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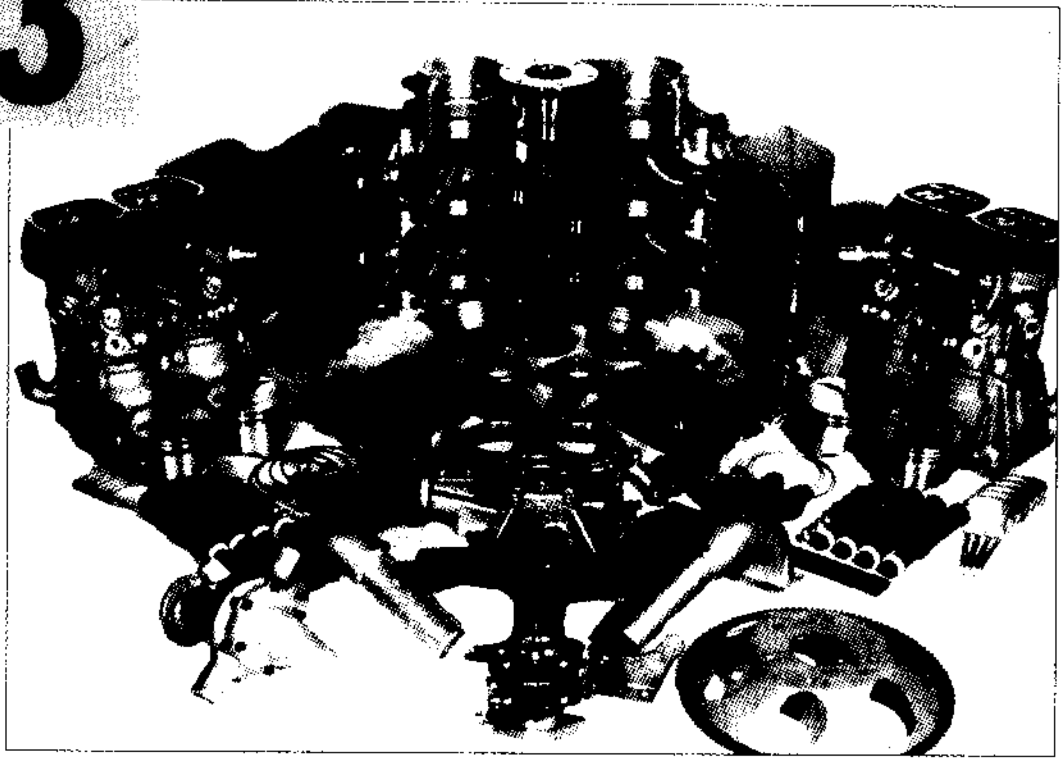


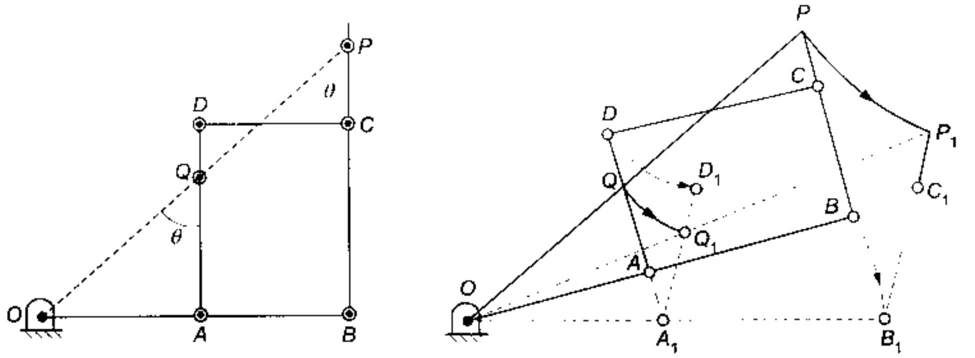
LOWER PAIRS

3.1 INTRODUCTION

In this chapter, we shall study the various mechanisms for the generation of straight and intermittent motion. These mechanisms play a vital role in generating the configurations for machines. The steering gears and Hooke's joint shall also be discussed.

3.2 PANTOGRAPH

This is a mechanism to produce the path traced out by a point on enlarged or reduced scale. Fig.3.1(a) shows the line diagram of a pantograph in which $AB = CD$. $BC = AD$ and $ABCD$ is always a parallelogram. OQP is a straight line. Point P describes a path similar to that described by Q. The pantograph is used as a copying mechanism.



(a) Original position (b) Displaced position

Fig.3.1 Pantograph

Proof

Triangles OAQ and OBP are similar because $\angle BOP$ is common.

$\angle AQO = \angle BPO$ are corresponding angles as $AQ \parallel BP$.

Hence, $$\frac{OA}{OB} = \frac{OQ}{OP} = \frac{AQ}{BP} \tag{3.1}$$

In the displaced position shown in Fig.3.1(b), as all links are rigid.

$$B_1O = BO,\ D_1A_1 = DA,\ A_1O = AO$$

and $$P_1B_1 = PB,\ B_1A_1 = BA,\ A_1Q_1 = AQ$$

Hence, $$\frac{OA_1}{OB_1} = \frac{A_1Q_1}{B_1P_1}$$

As $A_1B_1C_1D_1$ is a parallelogram, $A_1D_1 \parallel B_1C_1$, i.e. $A_1Q_1 \parallel B_1P_1$.

OQ_1P_1 is again a straight line so that Δs QA_1Q_1 and OB_1P_1 are similar.

$$\frac{OA_1}{OB_1} = \frac{OQ_1}{OP_1} \tag{3.2}$$

From (3.1) and (3.2), we get

$$\frac{OQ}{OP} = \frac{OQ_1}{OP_1} \quad \text{because} \quad OA = OA_1 \quad \text{and} \quad OB = OB_1.$$

Hence, QQ_1 is similar to PP_1 or they are parallel.

The pantograph is used in geometrical instruments in the manufacture of irregular objects, to guide cutting tools and as indicator rig for cross-head.

3.3 STRAIGHT LINE MOTION MECHANISMS

Now we shall study lower pairs generating straight line motion, intermittent motion and engine indicators. Straight line motion can be generated by either sliding pairs or turning pairs. Sliding pairs are bulky and get worn out rapidly. Straight line motion can be generated either accurately or approximately.

3.3.1 Accurate Straight Line Mechanisms

Mechanisms for accurate straight line motion are as follows:

1. Peaucellier mechanism
2. Hart mechanism
3. Scott–Russel mechanism.

Peaucellier mechanism A line diagram of the Peaucellier mechanism is shown in Fig.3.2.

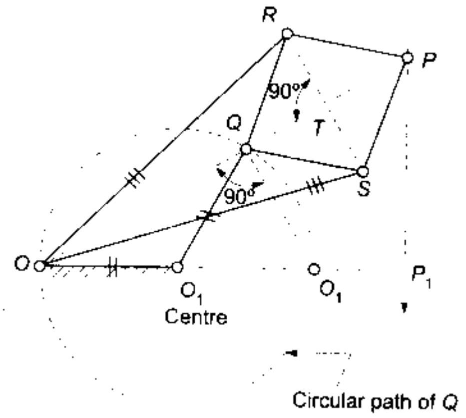


Fig.3.2 Peaucellier straight line mechanism

$OR = OS$ and $OO_1 = O_1Q$. Point P describes a straight line perpendicular to OO_1 produced.

Proof

Triangles ORQ and OSQ are congruent because $OR = OS$, $QR = QS$ and OQ is common. $\angle ROQ = \angle SOQ$. Also, $\angle OQR = \angle OQS$ and $\Delta PRQ \equiv \Delta PSQ$.

$$\angle OQR + \angle RQP = \angle OQS + \angle SQP = 180°$$

But OQ is a straight line.

$$\angle OQR + \angle RQP = \angle OQS + \angle SQP = 180°$$

Hence OQP is a straight line.

Now

$$\begin{aligned} OR^2 &= OT^2 + RT^2 \\ RP^2 &= RT^2 + TP^2 \\ OR^2 - RP^2 &= OT^2 - TP^2 \\ &= (OT + TP)(OT - TP) \\ &= OP \cdot OQ \end{aligned}$$

But OR and RP are always constant. Hence $OP \cdot OQ$ = constant

Draw $PP_1 \perp OO_1$ produced and join QQ_1. Triangles OQQ_1 and OPP_1 are similar, because $\angle OQQ_1 = \angle OP_1P = 90°$ and $\angle QOQ_1$ is common.

Hence, $OQ/OQ_1 = OP_1/OP$

or, $OQ \cdot OP = OQ_1 \cdot OP_1 = \text{constant}$

Now, $OQ_1 = 2OO_1 = \text{constant}$

Hence, $OP_1 = \text{constant}$

or point P moves in a straight line.

Hart mechanism The Hart mechanism is shown in Fig.3.3. OO_1 is a fixed link and O_1Q is the rotating link. Point Q moves in a circle with centre O_1 and radius O_1Q. Now, $ABCD$ is a trapezium so that $AB = CD$, $BC = AD$ and $BD \parallel AC$. Also, $BO/BA = BQ/BC = DP/DA$. Point P-describes a straight line perpendicular to OO_1 produced as Q moves in a circle with centre O_1.

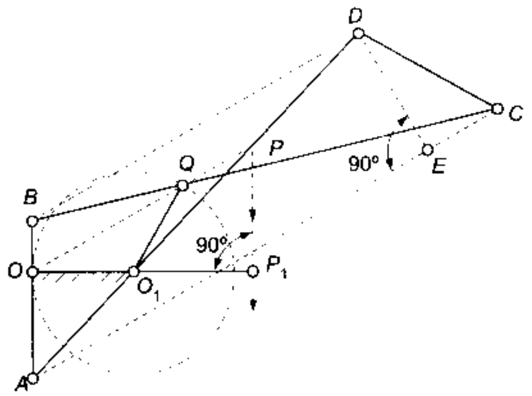


Fig.3.3 Hart straight line mechanism

Proof

In $\triangle ABD$, $BO/BA = PD/DA$. Hence $OP \parallel BD$.

Triagnles ABD and AOP are similar, because $\angle AOP = \angle ABD$ are corresponding angles and $\angle DAB$ is common.

$\therefore$ $AB/AO = BD/OP$

or $OP = BD \cdot AO/AB$ (3.3)

Similarly as $BO/AB = BQ/BC$. Hence $OQ \parallel AC$.

Now $AC \parallel BD$, $OQ \parallel BD$, $OP \parallel BD$. Hence $OP \parallel OQ$.

Since O is a common point, OQP is a straight line.

Now triangles BOQ and BAC are similar.

$\therefore$ $AC/OQ = AB/OB$

or $OQ = OB \cdot AC/AB$ (3.4)

From (3.3) and (3.4), we get

$$OP \cdot OQ = (BD \cdot AO/AB) \cdot (OB \cdot AC/AB)$$
$$= (AO \cdot OB/AB^2)(BD \cdot AC) \quad (3.5)$$

Draw $DE \perp AC$.

$$BD = AC - 2EC$$
$$AC = AE + EC$$
$$BD \cdot AC = (AC - 2EC)(AE + EC)$$
$$= (AE + EC - 2EC)(AE + EC)$$
$$= (AE - EC)(AE + EC)$$
$$= AE^2 - EC^2 \quad (3.6)$$

From $\triangle AED$, $AE^2 = AD^2 - DE^2$
From $\triangle CED$, $EC^2 = CD^2 - DE^2$

$$AE^2 - EC^2 = AD^2 - CD^2 \quad (3.7)$$

From (3.6) and (3.7), we get

$$BD \cdot AC = AD^2 - CD^2 \quad (3.8)$$
$$\therefore \quad OP \cdot OQ = (AO \cdot BO/AB^2)(AD^2 - CD^2$$
$$=$$ constant, as AO, $BO \cdot AB$, AD and CD are fixed.

Hence P describes a straight line perpendicular to OO_1 produced as point Q moves in a circle with centre O_1.

Scott–Russel mechanism The Scott–Russel mechanism is shown in Fig.3.4. This consists of a sliding pair and turning pairs. The Scott–Russel mechanism can be used to generate approximate and accurate straight lines.

1. When $OC = CP = CQ$, Q describes a straight line $QO \perp OP$, provided P moves in a straight line along OP.
2. If $CP \neq CQ$ then Q describes an approximate straight line perpendicular to OP, provided P moves along a straight line OP such that $OC = CP^2/CQ$.

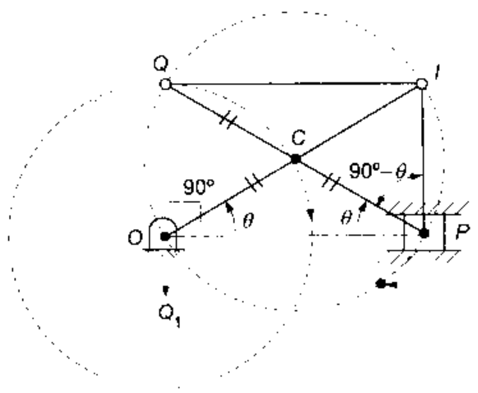


Fig.3.4 Scott–Russel straight line mechanism ($CP = CQ$)

Proof

1. As $OC = CP = CQ$, hence $\angle POQ = 90°$ and $OQ \perp OP$. If P moves along OP then Q moves along a line perpendicular to OP.

$$
\begin{aligned}
\text{Hence} \quad OP &= OC\cos\theta + CP\cos\theta = PQ\cos\theta \\
\angle CPI &= 90° - \theta \\
\text{Now} \quad \angle ICP &= \angle COP + \angle CPO = 2\theta \\
\text{Then} \quad \angle CIP &= 180° - \angle CPI - \angle ICP \\
&= 180° - 90° + \theta - 2\theta \\
&= 90° - \theta \\
\angle CIP &= \angle CPI \\
\text{or} \quad CI &= CP \\
\therefore \quad CP &= CQ = CO = CI \quad \text{and} \quad \angle POQ = 90°
\end{aligned}
$$

Hence OPIQ is a square. As the path of Q is $\perp QI$, I is the instantaneous centre. The point Q will move along a line perpendicular to OP.

2. When $CP \neq CQ$, it will form an elliptical trammel, as shown in Fig.3.5.

$$\frac{x^2}{CQ^2} + \frac{y^2}{CP^2} = 1$$

Because $x = CQ\cos\theta$ and $y = PC\sin\theta$

$$
\begin{aligned}
OC &= (\text{semi-minor axis})^2/\text{semi-major axis} \\
&= CP^2/CQ
\end{aligned}
$$

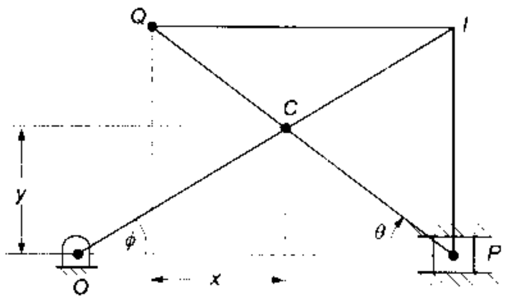


Fig.3.5 Scott–Russel mechanism when $CP \neq CQ$

As point C moves in a circle, for Q to move along an approximate straight line, $OC = CP^2/CQ$.

Limitations When $OC \perp OP$, P coincides with O and $OQ = 2OC$. Here a small displacement of P shall cause a large displacement of Q, requiring a relatively small displacement of P to give displacement to Q. This requires a highly accurate sliding surface.

3.3.2 Approximate Straight Line Mechanisms

Grasshopper mechanism The Grasshopper mechanism is shown in Fig.3.6. The crank OC rotates about a fixed point O. Point O_1 is a fixed pivot for link O_1P. For small angular displacements of O_1P, point Q on link PCQ will trace approximately a straight line path perpendicular to OP if

$$OC = \frac{CP^2}{CQ}$$

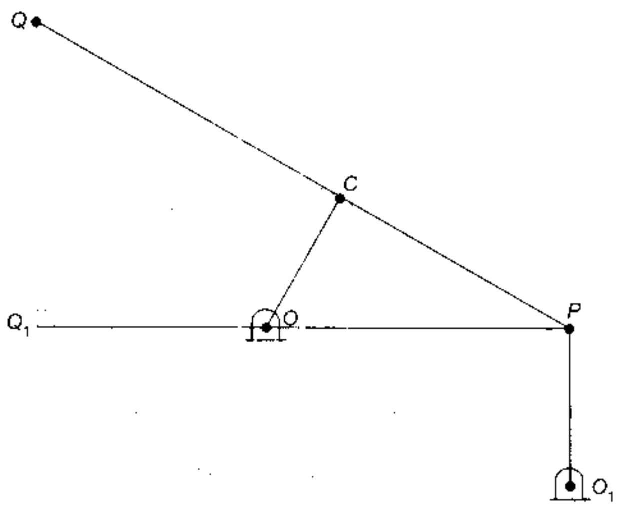
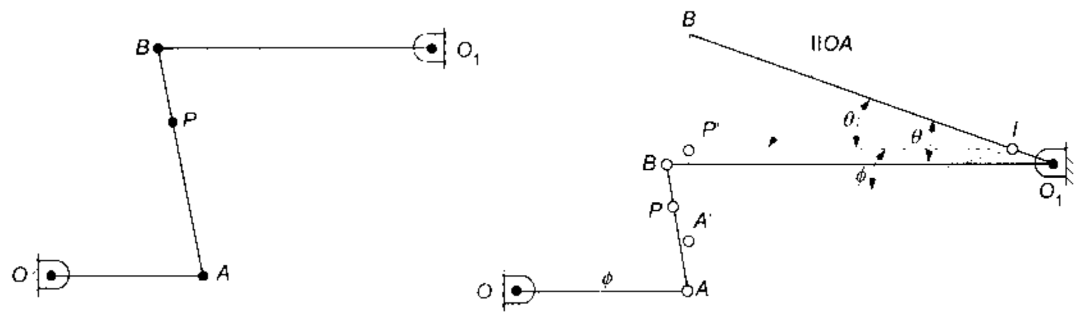


Fig.3.6 Grasshopper mechanism

Watt mechanism The Watt mechanism is shown in Fig.3.7(a).



(a) Watt mechanism (b) Watt mechanism in displaced position

Fig.3.7

Links OA and O_1B oscillates about O and O_1 respectively. Here, AB is a connecting link. Point P will trace an approximate straight line if $\frac{PA}{PB} \cong \frac{O_1B}{OA}$. In Fig.3.7 (b), θ and ϕ are the amplitudes of oscillation and I is the instantaneous centre of $A'B'$. Point P lies on the approximate straight line described by P.

Proof

$$\phi = \frac{AA'}{OA} \quad \text{and} \quad \theta = \frac{BB}{O_1B}$$

$$\therefore \quad \theta/\phi = \left(\frac{AA'}{OA}\right)\cdot\left(\frac{O_1B}{BB'}\right) \tag{3.9}$$

If $\quad \angle B'P'I = 90° \quad \text{then} \quad \sin\theta = \frac{B'P}{B'I}$

or $\quad \theta \approx \frac{B'P}{B'I} \quad \text{and} \quad \phi \approx \frac{A'P}{A'I}$

$$\phi/\theta \approx \left(\frac{A'P}{A'I}\right)\cdot\left(\frac{B'I}{B'P}\right) \tag{3.10}$$

From (3.9) and (3.10), we get

$$\left(\frac{AA}{OA}\right)\cdot\left(\frac{O_1B}{BB'}\right) \approx \left(\frac{A'P'}{A'I}\right)\cdot\left(\frac{B'I}{B'P'}\right)$$

Now $\quad \frac{AA}{BB'} \approx \frac{B'I}{A'I}$

Triangles OAA' and $IP'A'$ are approximately similar, as well as triangles O_1BB' and $B'P'I$ are approximately similar.

$$\therefore \quad \frac{O_1B}{OA} \approx \frac{A'P}{B'P'} \approx \frac{AP}{BP}$$

Therefore, P divides the coupler AB in the ratio of the lengths of oscillating links. Hence P will describe an approximate straight line for a certain position of its path.

Tchebicheff mechanism The Tchebicheff mechanism is shown in Fig.3.8. In this mechanism, $OA = O_1B$ and $AP = PB$. Here, P is the tracing point. Let $AB = 1$, $OA = O_1B = x$ and $OO_1 = y$.

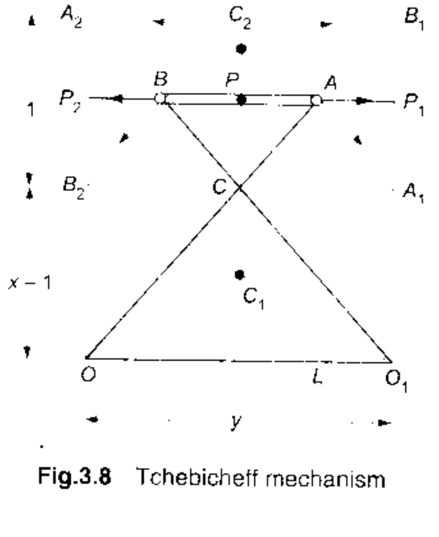


Fig.3.8 Tchebicheff mechanism

Now	$OB_2^2 = B_2O_1^2 - OO_1^2$
$\therefore$	$(x-1)^2 = x^2 - y^2$
or	$x = \dfrac{y^2+1}{2}$ (3.11)
Draw $AL \perp OO_1$, then	$OL = OO_1 - O_1L$
Also,	$O_1L = AP_1$
and	$AP_1 = \dfrac{P_2P1 - BA}{2} = \dfrac{y-1}{2}$
Again	$OL = OO_1 - O_1L = y - \dfrac{y-1}{2} = \dfrac{y+1}{2}$
Further	$OA^2 = AL^2 + OL^2$
	$x^2 = \left[x - \dfrac{1}{2}\right]^2 + \left[y + \dfrac{1}{2}\right]^2$
or	$x = \dfrac{y^2}{4} + \dfrac{1}{2} + \dfrac{y}{2}$ (3.12)

From (3.11) and (3.12), we get

$$\frac{y^2+1}{2} = \frac{y^2}{4} + \frac{1}{2} + \frac{y}{2}$$

or

$$y = 2$$

Substituting in (3.11), we find that, $x = 3.5$.

That is $AB : OO_1 : OA :: 1 : 2 : 3.5$

P moves horizontally because instantaneous centre of AB will lie on point C which is the intersection of OA and BO_1. Thus C lies just below P.

Robert's mechanism This mechanism is shown in Fig.3.9. The lengths of links are such that $AB = CD$, $BE = EC$ and EF is perpendicular to BC. Here, F is the tracing point. When the mechanism is displaced (shown by dotted lines), the point F will approximately trace a straight line parallel to BC. Produce AB' and DC' to meet at I, the instantaneous centre of link BC. From I drop a vertical line to intersect $E'F'$ at F'. The velocity v_f of point F' is perpendicular to the line joining I and F' and is thus a horizontal line.

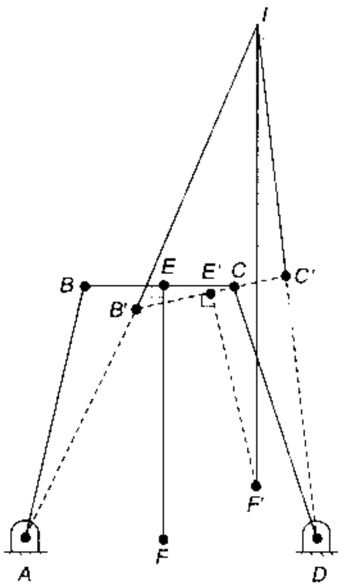


Fig.3.9 Robert's mechanism

3.4 INTERMITTENT MOTION MECHANISMS

These mechanisms are used to convert continuous motion into intermittent motion. The mechanisms used for this purpose are the Geneva wheel and the ratchet mechanism.

Geneva wheel The Geneva wheel as shown in Fig.3.10, consists of a plate 1 which rotates continuously and contains a driving pin P that engages in a slot in the driven member 3. Member 2 is turned $\frac{1}{4}$th of a revolution for each revolution of plate 1. The slot in member 2 must be tangential to the path of pin upon engagement in order to reduce shock. The angle β is half the angle turned through by member 2 during the indexing period. The locking plate serves to lock member 2 when it is not being indexed. Cut the locking plate back to provide clearance for member 2 as it swings through the indexing angle. The clearance arc in the locking plate will be equal to twice the angle α.

Ratchet mechanism This mechanism is used to produce intermittent circular motion from an oscillating or reciprocating member. Fig.3.11 shows the details of a ratchet mechanism. Wheel 4 is given intermittent circular motion by means of arm 2 and driving pawl 3. A second pawl 5 prevents 4 from turning backward when 2 is rotated clockwise in preparation for another stroke. The line of action PN of the driving pawl and tooth must pass between centers O and A in order to have the pawl 3 remain in contact with the tooth. This mechanism is used particularly in counting devices.

3.5 PARALLEL LINKAGES

These are the four-bar linkages in which the opposite links are equal in length and always form a parallelogram. There are three types of parallel linkages: parallel rules, universal drafting machine and lazy tongs. They are used for producing parallel motion.

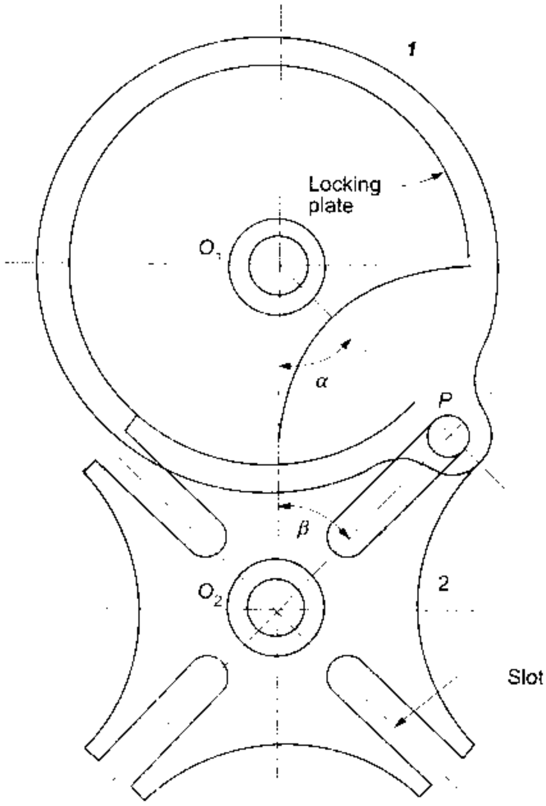


Fig.3.10 Geneva wheel

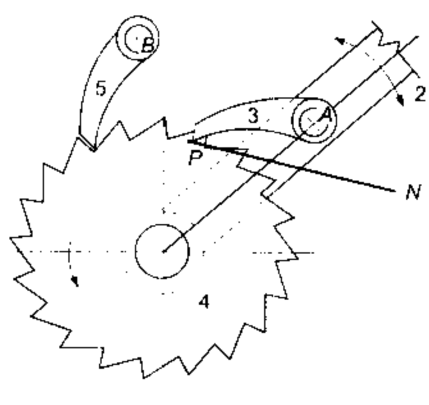


Fig.3.11 Ratchet mechanism

Parallel rules A parallel rule is shown in Fig.3.12, in which $AB = CD = EF = GH = IJ$ and $AC = BD$, $CE = DF$, $EG = FH$, $GI = HJ$. Here AB, C, EF, GH and IJ will always be parallel to each other.

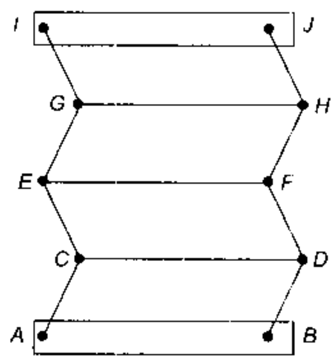


Fig.3.12 Parallel rule

Universal drafting machine A universal drafting machine is shown in Fig.3.13, in which $AB = CD$; $AC = BD$; $EF = GH$; and $EG = FH$. Position of points A and B are fixed. Similarly the positions of points E and F are fixed with respect to C and D. The positions of scales I and II are fixed with respect to points G and H. Then $ABDC$ is a parallelogram. Line CD will always be parallel to AB so that the direction of CD is fixed. Therefore, the direction of EF is fixed. Further $EFHG$ is a parallelogram, so GH is always parallel to EF such that the direction of GH is also fixed, whatever their actual position may be.

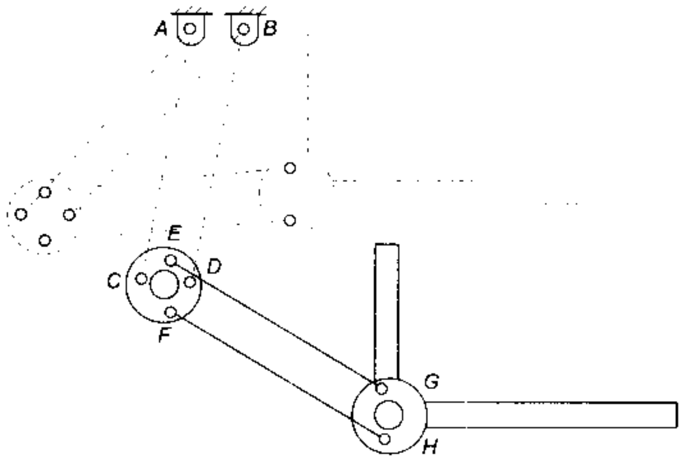


Fig.3.13 Universal drafting machine

Lazy tongs Lazy tongs as shown in Fig.3.14, consists of pin joint A attached to a fixed point. Point B moves on a roller. All other joints are pin jointed. The vertical movement of the roller affects the movement of the end C to move in a horizontal direction. It is used to support a telephone or a bulb at point C for horizontal movements.

3.6 ENGINE PRESSURE INDICATORS

A pressure indicator is an instrument used to obtain a graphical record of the pressure-volume diagram of a reciprocating engine. It consists of a cylinder with a piston, a straight line motion mechanism with a pencil and a drum with paper wrapped around it. The indicator cylinder is connected to the engine cylinder which causes the movement of the indicator piston with change of pressure in the engine cylinder. The piston motion is constrained by a spring so that the piston displacement is a direct measure of the working fluid pressure acting upon it. The displacement and hence volume is then recorded by the pencil with the help of straight line mechanism on the drum paper. The requirements of the straight line mechanism are as follows:

1. The pencil point should move in a straight line parallel to the axis of the indicator piston.
2. The velocity ratio of the indicator piston and pencil should be constant.
3. The indicator piston motion should be magnified so as to get a good size indicator diagram.
4. The friction should be the least.

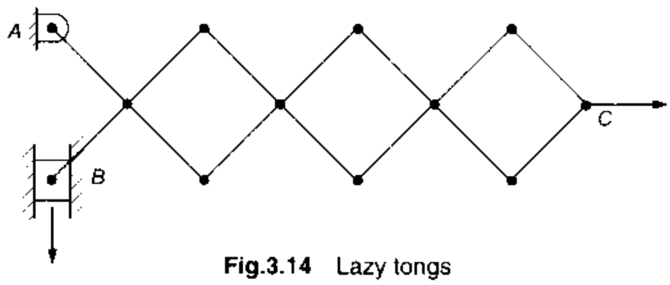


Fig.3.14 Lazy tongs

3.6.1 Types of Indicators

The various types of indicators are simplex, Crosby, Richards, Thomson and Dobbie–McInnes.

Simplex indicator The simplex indicator (Fig.3.15) employs pantograph mechanism. Point *Q* on link *AD* coincides with *D* and *P* is a point on *BC* produced such that *OQP* is a straight line. *ABCD* form a parallelogram with all joints pin jointed. Point *Q* lies on the piston of the indicator. The pencil to record the indicator diagram is fixed at point *P* which describes a path similar to that of *Q*.

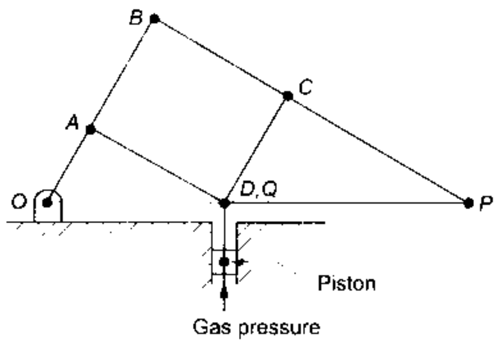


Fig.3.15 Simplex indicator

Crosby indicator The Crosby indicator (Fig.3.16) employs the modified form of pantograph to generate motion of pencil point *P* similar to that of point *Q* lying on the indicator piston. The following conditions are to satisfied by this mechanism:

1. Velocity ratio between P and Q is constant, i.e. $\frac{v_p}{v_q}$ = constant.
2. Point P travels along a straight line.

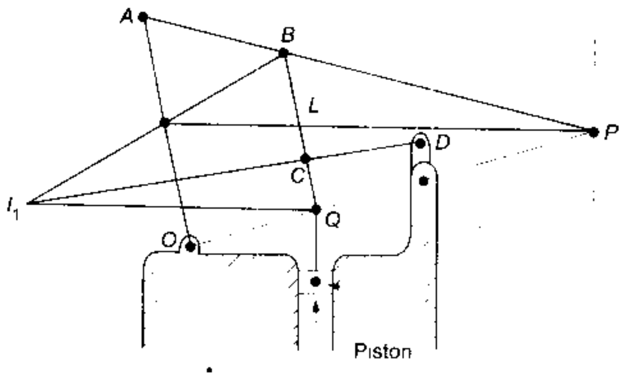


Fig.3.16 Crosby indicator

Proof

Draw instantaneous centres I_1 and I_2 of links QB and AP, respectively. I_1 is obtained by drawing a horizontal line from Q which meets the line DC produced in I_1. For I_2, draw a horizontal line from P meeting OA in I_3. The line I_2P cuts the link QB in I_1. The point I_2 will lie in I_1B.

Now $$\frac{v_b}{v_q} = \frac{I_1B}{I_1Q}$$

but $$\frac{I_1B}{I_1Q} = \frac{I_2B}{I_2L}$$

$\therefore$ $$\frac{v_b}{v_q} = \frac{I_2B}{I_2L} \tag{3.13}$$

Also $$\frac{v_p}{v_b} = \frac{I_2P}{I_2B} \tag{3.14}$$

Multiplying (3.13) and (3.14), we get

$$\frac{v_p}{v_q} = \frac{I_2P}{I_2L}$$

As OA is parallel to QB or I_2A is parallel to BL, Δs PI_2A and PLB are similar.

$\therefore$ $$\frac{I_2P}{I_2L} = \frac{AP}{AB}$$

$$\frac{v_p}{v_q} = \frac{AP}{AB} = \text{constant}$$

Since lengths AP and AB are fixed.

Richard indicator The Richard indicator (Fig.3.17) employs Watt mechanism $OABO_1$ to guide the pencil point at P. The motion to link AC is given by the piston rod of the indicator piston at C through link QC. If a line QD is drawn parallel to OCA, then $OCADQ$ forms a pantograph, having point P on link AD produced.

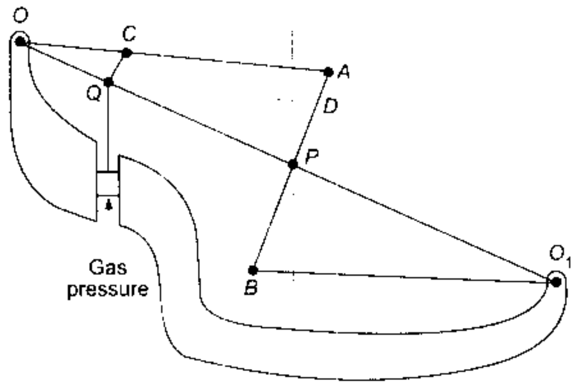


Fig.3.17 Richard indicator

Thomson indicator The Thomson indicator (Fig.3.18) employs Grasshopper mechanism $OABO_1$, the tracing point P lying on link AB produced. Link AB gets the motion from the piston rod of the indicator at C which is connected by link QC at Q to the end of indicator piston rod. Link QC is parallel to link OA.

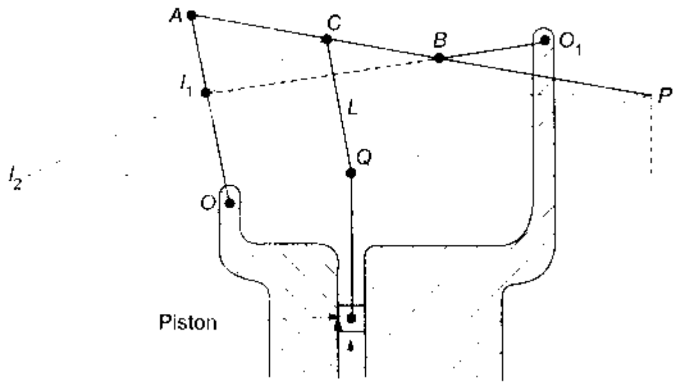


Fig.3.18 Thomson indicator

The velocity ratio $\frac{v_p}{v_q}$ = constant. To prove this, draw the instantaneous centres I_1 and I_2 of the links AB and QC respectively. I_1P cuts CQ in L.

Proof

$$\frac{v_c}{v_q} = \frac{I_2C}{I_2Q}$$

Now triangles I_1CL and I_2CQ are similar. Therefore

$$\frac{I_2C}{I_2Q} = \frac{I_1C}{I_1L}$$

Also
$$\frac{v_p}{v_c} = \frac{I_1P}{I_1C}$$

$$\therefore \qquad \frac{v_p}{v_q} = \frac{I_1P}{I_1L}$$

Δs PAI_1 and PCL are similar. Hence

$$\frac{I_1P}{I_1L} = \frac{AP}{AC}$$

$$\therefore \qquad \frac{v_p}{v_q} = \frac{AP}{AC} = \text{constant}$$

Since the lengths AP and AC are fixed

Dobbie–McInnes indicator In the Dobbie–McInnes indicator (Fig.3.19), the motion is given to link O_1B by the link QC connected to the indicator piston and I_1 and I_2 are the instantaneous centres of AB and QC, respectively. The line I_1P cuts QC in L. Draw $BM \perp I_1P$ from point B. OQP is a straight line. The ratio $\frac{v_p}{v_q}$ = constant.

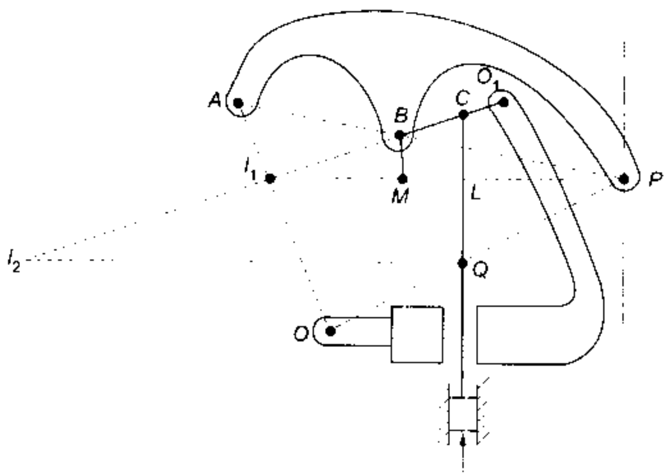


Fig.3.19 Dobbie-McInnes indicator

Proof

$$\frac{v_c}{v_q} = \frac{I_2C}{I_2Q}$$

Triangles I_1BM, I_1CL and I_1CQ are similar.

$$\frac{I_2C}{I_2Q} = \frac{I_1C}{I_1L} = \frac{I_1B}{I_1M}$$

$$\frac{v_c}{v_q} = \frac{I_1B}{I_1M}$$

Now

$$\frac{v_b}{v_c} = \frac{O_1B}{O_1C}$$

Also

$$\frac{v_p}{v_b} = \frac{I_1P}{I_1B}$$

$$\therefore \quad \frac{v_p}{v_q} = \left(\frac{I_1P}{I_1M}\right)\cdot\left(\frac{O_1B}{O_1C}\right)$$

In similar triangles PBM and PAI_1,

$$\frac{I_1P}{I_1M} = \frac{AP}{AB}$$

$$\therefore \quad \frac{v_p}{v_q} = \frac{PA}{AB}\cdot\frac{O_1B}{O_1C} = \text{constant}$$

Since the lengths of all four links PA, AB, O_1B and O_1C are fixed

Example 3.1

A circle with AB as diameter has a point C on its circumference. The point D is a point on AC produced such that if C turns about A and $AC \cdot AD$ is constant. Prove that the point D moves in a straight line perpendicular to AB.

■ Solution

Let D_1D be perpendicular to AB produced, as shown in Fig.3.20.

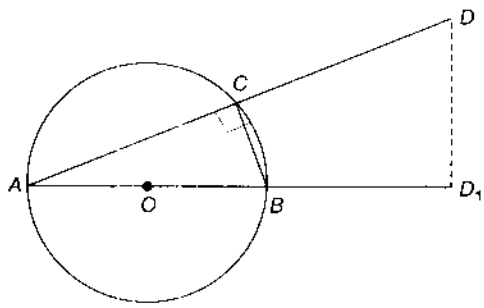


Fig.3.20 Mechanism with circle

Now $\angle ACB = 90°$, being the angle in a semicircle. Also $\angle AD_1D = 90°$. Therefore, triangles ACB and AD_1D are similar, as $\angle CAB$ is common.

$$\frac{AC}{AB} = \frac{AD_1}{AD}$$

or

$$AB\cdot AD_1 = AC\cdot AD$$

or

$$AD_1 = \frac{AC\cdot AD}{AB} = \text{constant}$$

Since AB is fixed and $AC \cdot AD$ = constant

Thus AD_1 will be constant for all positions of C. Therefore, the location of D_1 is fixed, which means that D moves in a straight line perpendicular to AB.

Example 3.2

In a Grasshopper mechanism, shown in Fig.3.21, $OC = 100$ mm, $PC = 150$ mm, and $PQ = 375$ mm. Determine the magnitude of the vertical force at Q necessary to resist a torque of 120 N m. applied to the link OC when it makes an angle of 30° with the horizontal.

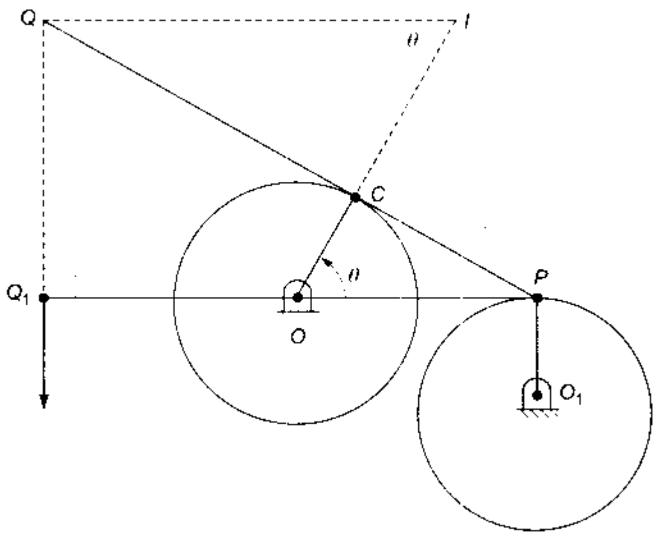


Fig.3.21 Grasshopper mechanism

Solution

$$QC = PQ - PC = 375 - 150 = 225 \text{ mm}$$

$$\frac{OC}{PC} = \frac{PC}{QC}$$

or

$$\frac{100}{150} = \frac{150}{225}$$

Thus the condition for the dimensions of a Grasshopper mechanism is satisfied, and point Q will approximately trace a straight line perpendicular to OP.

Now

$$F_q \cdot v_q = T_c \cdot \omega_c$$

or

$$F_q = \frac{T_c \cdot v_c}{v_q \cdot OC}$$

The instantaneous centre of PQ is at I. Let $\angle QIC = \theta$. Also triangles QIC and OCP are similar.

$$\frac{v_c}{v_q} = \frac{IC}{IQ} = \frac{OC}{OP}$$

Thus

$$F_q = \frac{T_c}{OP}$$

When = 30°,

$$OP = 100\cos 30° + \left[(150)^2 - (100\sin 30°)^2\right]^{0.5}$$

$$= 228 \text{ mm}$$

$$F_q = \frac{120}{0.228} = 526.3 \text{ N}$$

3.7 AUTOMOBILE STEERING GEAR MECHANISMS

Steering gears are used in vehicles to control their direction of motion. We shall study the relationship between the various parameters of the steering gears for correct steering.

3.7.1 Fundamental Equation for Correct Steering

For correct steering, the instantaneous centre of the front and rear wheels must lie at the same point. For the steering gear mechanism shown in Fig.3.22, *AC* and *BD* are stub axles,

where, $AE = l$ = wheel base

$CD = a$ = wheel track

$AB = b$ = distance between the pivots of front axles

I = common instantaneous centre of all wheels

Now
$$b = AP - BP$$
$$= l(\cot\phi - \cot\theta)$$

or
$$\cot\phi - \cot\theta = \frac{b}{l} \quad (3.15)$$

This represents the fundamental equation for correct steering.

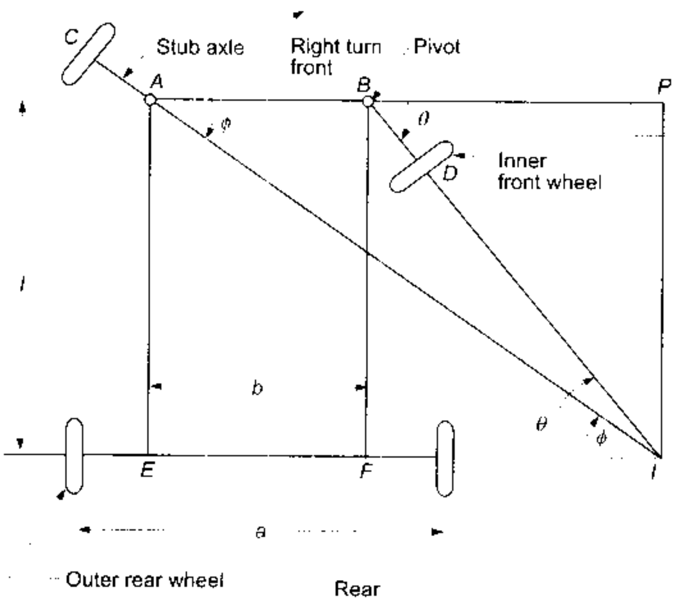


Fig.3.22 Automobile steering gear

3.7.2 Steering Gears

A steering gear is a mechanism for automatically adjusting values of θ and ϕ for correct steering. There are two types of steering gears commonly used in automobiles:

1. Davis steering gear—which has sliding pairs.
2. Ackermann steering gear—which has turning pairs.

Davis steering gear A line diagram of the Davis steering gear is shown in Fig.3.23.

where,

KL = cross link which slides parallel to AB
CAK, DBL = bell crank levers
E, F = bearings
K, L = pins with slide blocks
AC, BD = stub axles
A, B = pivots

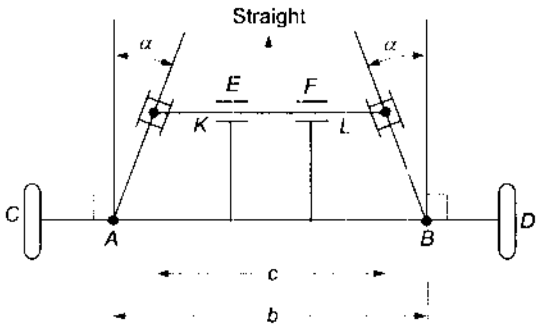


Fig.3.23 Davis steering gear for straight drive

Determination of angle α The Davis steering gear is shown in Fig.3.24 when the automobile is turning right. In this position, we have

$$y = AK\sin\alpha = BL\sin\alpha$$

$$\tan(\alpha - \theta) = \frac{y - x}{h}$$

or
$$\frac{\tan\alpha - \tan\theta}{1 + \tan\alpha\tan\theta} = \frac{y - x}{h}$$

But
$$\tan\alpha = \frac{y}{h}$$

∴
$$\frac{\frac{y}{h} - \tan\theta}{1 + (y/h)\cdot\tan\theta} = \frac{y - x}{h}$$

or
$$\tan\theta = \frac{xh}{y^2 - xy + h^2} \quad (3.16)$$

Similarly
$$\tan(\alpha + \phi) = \frac{y + x}{h} \quad (3.17)$$

so that
$$\tan\phi = \frac{xh}{y^2 + xy + h^2}$$

From (3.16) and (3.17), we get
$$\cot\phi - \cot\theta = y^2 + xy + h^2 - y^2 + xy - \frac{h^2}{xh} = \frac{b}{l}$$

or
$$\frac{2y}{h} = \frac{b}{l}$$

or
$$\tan\alpha = \frac{b}{2l} \quad (3.18)$$

Generally $\frac{b}{l} = 0.4$ to 0.5 so that $\alpha = 11.3°$ to $14.1°$.

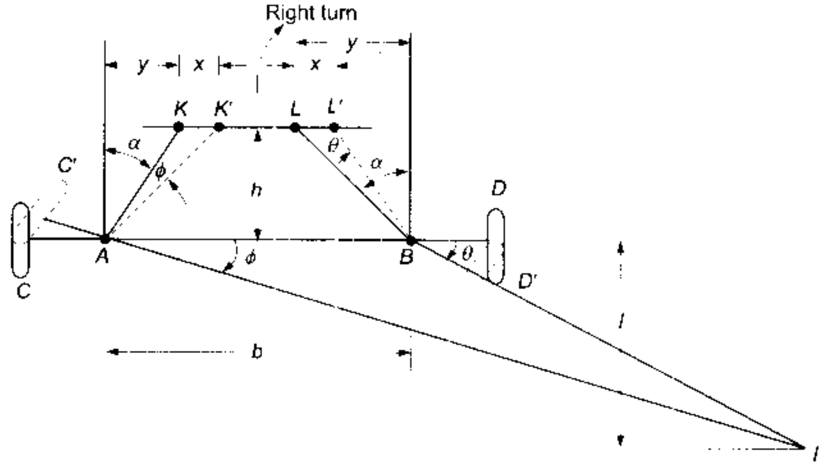


Fig.3.24 Davis steering gear taking a right turn

Ackermann steering gear The Ackermann steering gear is shown in Fig.3.25.

where, $ABLK$ = four-bar linkage
CAK, DBL = bell crank levers
AK, BL = short arms
KL = track rod
l = wheel base

For correct steering, $\cot\phi - \cot\theta = \frac{b}{l}$

Generally, $\frac{b}{l} = 0.4$ to $0.5 \approx 0.455$.

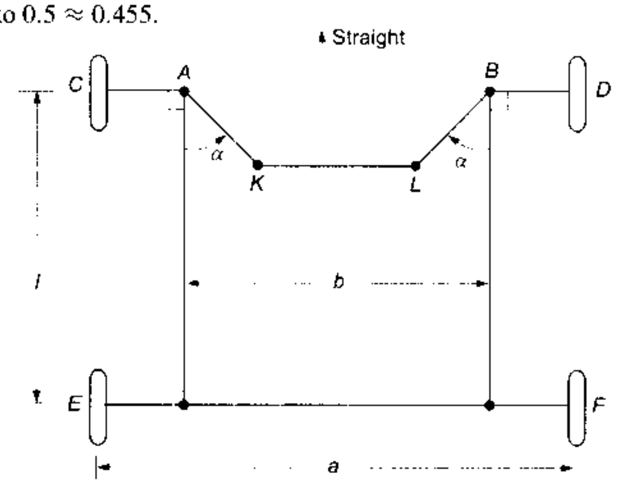
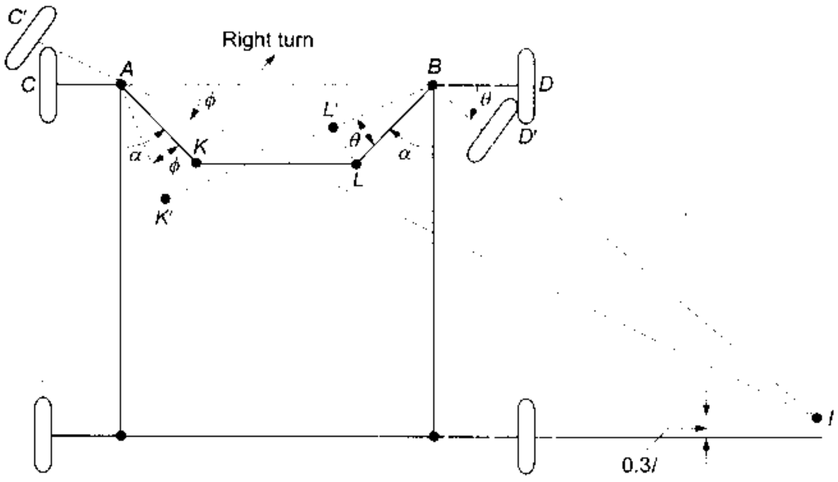
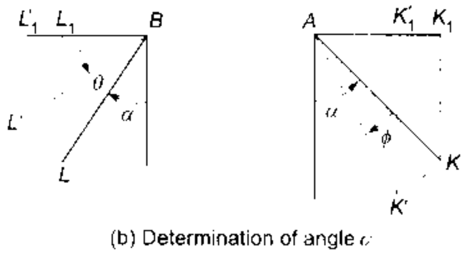


Fig.3.25 Ackermann steering gear for straight drive

Determination of angle α The displaced position of the steering gear is shown in Fig.3.26(a) when the automobile is taking a right turn. The instantaneous centre I lies at a distance of $0.3l$ from the rear axis.



(a) Ackermann steering gear taking a right turn



(b) Determination of angle α

Fig.3.26

From Fig.3.26(b), we have

$$\text{Projection of arc } K'K \text{ on } AB = \text{Projection of arc } L'L \text{ on } AB$$

$$K_1K_1' = L_1L_1$$

$$AK[\sin\alpha - \sin(\alpha - \phi)] = BL[\sin(\alpha + \theta) - \sin\alpha]$$

Now $$AK = BL$$

$$\sin\alpha - \sin\alpha\cos\phi + \cos\alpha\sin\phi = \sin\alpha\cos\theta + \cos\alpha\sin\theta - \sin\alpha$$

$$\sin\alpha(2 - \cos\phi - \cos\theta) = \cos\alpha(\sin\theta - \sin\alpha)$$

$$\tan\alpha = \frac{\sin\theta - \sin\phi}{2 - \cos\phi - \cos\theta} \quad (3.19)$$

Graphical method to determine α Draw a horizontal line OX (Fig.3.27). Make $\angle XOQ = \theta$ and $\angle XOP = \phi$. With any radius, draw arc PRQ. Join PR and produce it so that $PR = RM$. Join MQ and produce. Draw $ON \perp NQM$. Then $\alpha = 90° - \angle XON$.

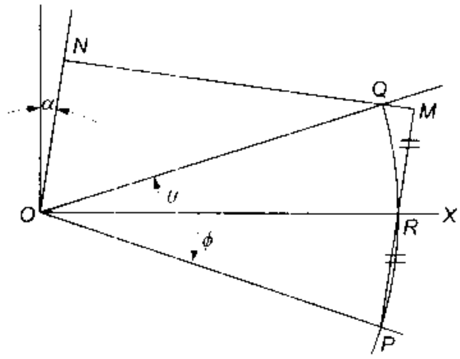


Fig.3.27 Graphical method to determination angle α

Proof

In Fig.3.28, let $BL = x$ so that $b = c + 2x \sin\alpha$.

For correct steering,

$$b = l(\cot\phi - \cot\theta)$$
$$= c\cos\psi + x\sin(\alpha+\theta) + x\sin(\alpha-\phi)$$

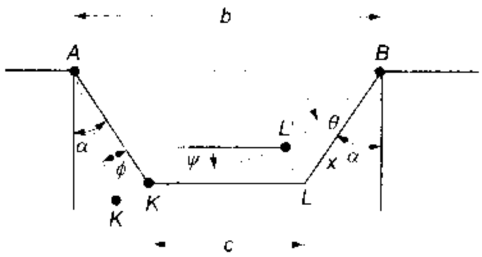


Fig.3.28 Graphical method to determine angle α

If ψ is small, $c\cos\psi = c$.

$$c + 2x\sin\alpha = c + x\sin(\alpha+\theta) + x\sin(\alpha-\phi)$$

or

$$2\sin\alpha = \sin(\alpha+\theta) + \sin(\alpha-\theta)$$

In Fig.3.29, let $r = OP = OR = OQ$. In right angled triangle OQN,

$$\angle NOQ = 90° - \alpha - \theta = 90 - (\alpha+\theta)$$
$$ON = r\cos[90° - (\alpha+\theta)] = r\sin(\alpha+\theta)$$

Draw $RL \perp ON$ then $RL \parallel SN$.

i.e. $$\angle SRQ = \angle\alpha$$

Also $$\angle ORL = \angle\alpha$$
$$OL = r\sin\alpha$$

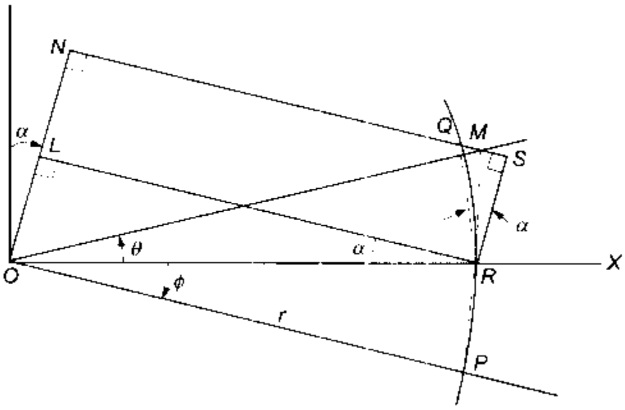


Fig.3.29 Graphical method to determine angle α

Now $NL = RS$. In $\triangle OPR$,

$$\frac{PR}{\sin\phi} = \frac{r}{\sin(90° - 0.5\phi)}$$

or

$$PR = \frac{r\sin\phi}{\cos(0.5\phi)}$$

$$= 2r\sin(0.5\phi)$$

From triangle RSM in Fig.3.30, we have

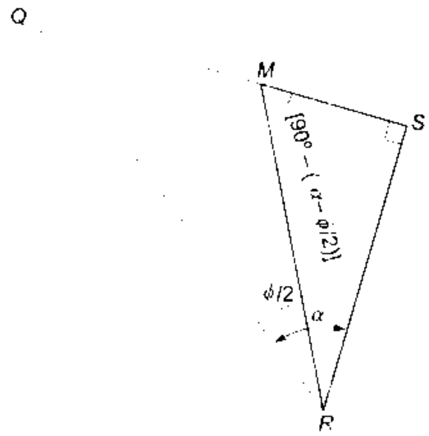


Fig.3.30 Graphical method to determine angle α

$$\angle MRS = \alpha - 0.5\phi$$

$$\angle SMR = 90° - (\alpha - 0.5\phi)$$

Also

$$\frac{RS}{RM} = \cos(\alpha - 0.5\phi)$$

Now $RM = PR$.

$$\frac{RS}{PR} = \cos(\alpha - 0.5\phi)$$

or
$$RS = 2r\sin(0.5\phi)\cos(\alpha - 0.5\phi)$$
$$= r\sin\alpha - r\sin(\alpha - \phi)$$
$$ON = OL + LN = OL + RS$$
$$r\sin(\alpha + \theta) = r\sin\alpha + r\sin\alpha - r\sin(\alpha - \phi)$$
or
$$2\sin\alpha = \sin(\alpha + \theta) + \sin(\alpha - \phi) \quad (3.20a)$$
or
$$\tan\alpha = \frac{(\sin\theta - \sin\phi)}{(2 - \cos\theta - \cos\phi)} \quad (3.20b)$$

Hence proved.

Example 3.3

In a Davis steering gear, the distance between the pivots of the front axle is 1 m and the wheel base is 3.5 m. When the automobile is moving along a straight path, find the inclination of the track arms to the longitudinal axis of the automobile?

■ **Solution**

$$\text{Here } b = 1 \text{ m} \quad \text{and} \quad l = 3.5 \text{ m}$$
$$\tan\alpha = \frac{b}{2l}$$
$$= \frac{1}{5} = 0.2$$
$$\alpha = 11.31°$$

Example 3.4

A car with a track of 1.5 m and a wheel base of 3.9 m has a steering gear mechanism of the Ackermann type. The distance between the front stub axle pivots is 1.3 m. The length of each track is 150 mm and the length of track rod is 1.2 m. Find the angle turned through by the outer wheel if the angle turned through by the inner wheel is 30°.

■ **Solution**

Here $a = 1.5$ m, $l = 3.9$ m, $b = 1.3$ m, $c = 1.2$ m, $AK = 0.15$ mm.

$$\cot\phi - \cot\theta = \frac{b}{l}$$
$$= \frac{1.3}{3.9} = 0.44827$$
$$\cot\phi - \cot 30° = 0.44827$$
$$\cot\phi - 1.73205 = 0.44827$$
$$\cot\phi = 3.18032$$
$$\phi = 24.64°$$

3.8 HOOKE'S JOINT OR UNIVERSAL COUPLING

Hooke's joint is a device that connects two shafts whose axes are neither coaxial nor parallel but intersect at a point. This is used to transmit power from the engine to the rear axle of an automobile and other similar applications.

Hooke's joint, as shown in Fig.3.31, consists of two forks connected by a centre piece, having the shape of a cross or square carrying four trunnions. The ends of the two shafts to be connected together are fitted to the forks.

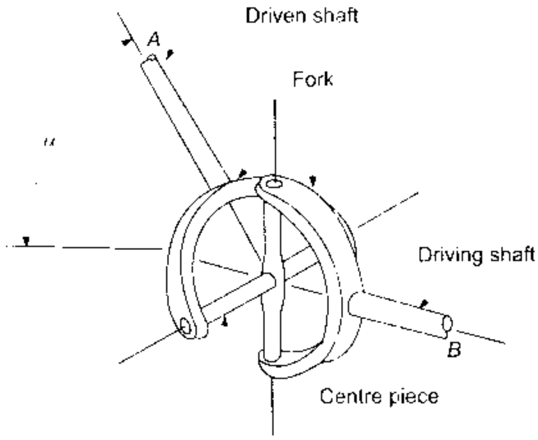


Fig.3.31 Hooke's joint

3.8.1 Velocities of Shafts

Let the driving shaft A and the driven shaft B be connected by a joint having two forks KL and MN. The two forks are connected by a cross $KLMN$ intersecting at O, as shown in Fig.3.32. Let the angle between the axes of the shafts be α. Let fork KL move through an angle θ in a circle to the position K_1L_1 in the front view. The fork MN will also move through the same angle θ. As MN is not in the same plane, it moves in an ellipse in the front view to its new position M_1N_1. To find the true angle, project M_1 to the top view which cuts the horizontal axis in R and fork MN in R_1. Rotate R_1 to R_2 on the horizontal axis with centre O. Project R_2 back in the front view cutting the circle in M_3. Join OM_3. Measure MOM_2, which is the true angle ϕ. Thus when the driving shaft A revolves through an angle θ, the driven shaft B will revolve through an angle ϕ.

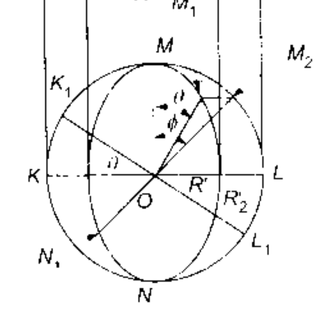


Fig.3.32 Hooke's joint analysis

Now $$\tan\phi = \frac{OR'_2}{R'_2M_2}$$

and $$\tan\theta = \frac{OR'}{R'M_1}$$

$\therefore$ $$\frac{\tan\phi}{\tan\theta} = \frac{OR'_2}{R'_2M_2}\cdot\frac{R'M_1}{OR'}$$

But $$R'M_1 = R_2'M_2$$

$\therefore$ $$\frac{\tan\phi}{\tan\theta} = \frac{OR'_2}{OR'} = \frac{OR_1}{OR}$$

Now $$OR = OR_1\cos\alpha$$

$$\frac{\tan\phi}{\tan\theta} = \frac{1}{\cos\alpha}$$

or $$\tan\theta = \cos\alpha\tan\phi \tag{3.21}$$

Angular velocity of driving shaft, $$\omega_A = \frac{d\theta}{dt}$$

Angular velocity of driven shaft, $$\omega_B = \frac{d\phi}{dt}$$

Differentiating (3.21), we get

$$\frac{\sec^2\theta\cdot d\theta}{dt} = \frac{\cos\alpha\sec^2\phi\, d\phi}{dt}$$

$$\frac{d\phi/dt}{d\theta/dt} = \frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta}$$

or $$\frac{\omega_B}{\omega_A} = \frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta}$$

Case (i) $\dfrac{\omega_B}{\omega_A} = 1$

$$\cos\alpha = 1-\sin^2\alpha\cos^2\theta$$

or $$\cos\theta = \pm\sqrt{1/(1+\cos\alpha)}$$

This condition occurs once in each quadrant, as shown in Fig.3.33.

Case (ii) $\dfrac{\omega_B}{\omega_A}$ is minimum when the denominator is maximum, that is $\sin^2\alpha\cos^2\theta$ must be minimum or $\cos\theta = 0°$. Thus $\dfrac{\omega_B}{\omega_A}$ is minimum at $\theta = 90°$ or $270°$. Then

$$\frac{\omega_B}{\omega_A} = \cos\alpha \tag{3.22}$$

Case (iii) $\dfrac{\omega_B}{\omega_A}$ is maximum when denominator is minimum, that is, when $\cos^2\theta = 1$ or $\cos\theta = \pm 1$, i.e. $\theta = 0°$ or $180°$. Then

$$\frac{\omega_B}{\omega_A} = \frac{1}{\cos\alpha} \tag{3.23}$$

Case (iv) Maximum variation of velocity of driven shaft

$$= \frac{(\omega_B)_{max} - (\omega_B)_{min}}{(\omega_B)_{mean}}$$

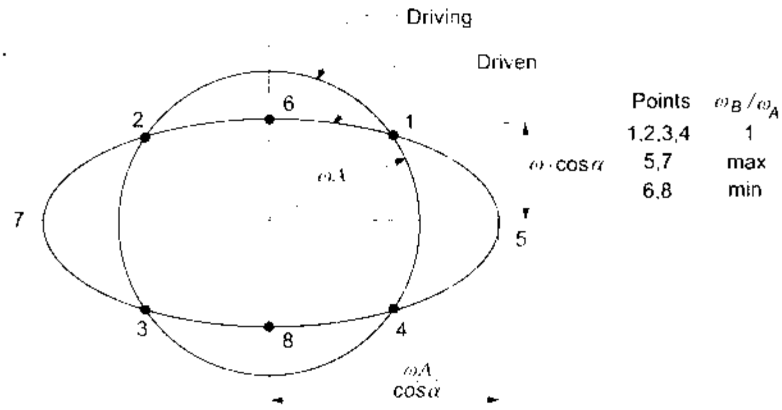


Fig.3.33 Polar velocity diagram for Hooke's joint

Now $(\omega_B)_{mean} = \omega_A$

$$\text{Maximum speed variation} = \frac{\left(\frac{\omega_A}{\cos\alpha}\right) - \omega_A \cos\alpha}{\omega_A}$$

$$= \frac{1 - \cos^2\alpha}{\cos\alpha}$$

$$= \tan\alpha \sin\alpha$$

For α to be small,

$$\tan\alpha \approx \alpha \quad \text{and} \quad \sin\alpha \approx \alpha$$

$$\text{Maximum speed variation} \approx \alpha^2$$

3.8.2 Angular Acceleration of Driven Shaft

$$\frac{\omega_B}{\omega_A} = \frac{\cos\alpha}{1 - \sin^2\alpha\cos^2\theta}$$

$$\frac{d\omega_B}{dt} = \omega_A \cdot d\theta/dt \cdot \cos\alpha \left[\frac{2\sin^2\alpha \cdot \cos\theta \cdot \sin\theta}{(1 - \sin^2\alpha\cos^2\theta)^2}\right]$$

Acceleration of the driven shaft,

$$\alpha_B = \frac{\omega_A^2 \cos\alpha \sin^2\alpha \sin 2\theta}{(1 - \sin^2\alpha\cos^2\theta)^2}$$

For acceleration to be maximum or minimum, $\frac{d\alpha_B}{d\theta} = 0$.

$$\frac{d\alpha_b}{d\theta} = \omega_A^2 \cos\alpha \left[\frac{\sin^2\alpha\, 2\sin 2\theta(1 - \sin^2\alpha\cos^2\theta)\sin^2\alpha\sin 2\theta - (1-\sin^2\alpha\cos^2\theta)^2 \cdot 2\cos 2\theta}{(1-\sin^2\alpha\cos^2\theta)^4} \right]$$

$$= \cos^2 2\theta + \left(\frac{2}{\sin^2\alpha - 1}\right)\cos 2\theta - 2$$

$$= 0$$

or $$\cos 2\theta \approx \frac{2\sin^2\alpha}{2-\sin^2\alpha} \tag{3.27}$$

3.9 DOUBLE HOOKE'S JOINT

The double Hooke's joint, as shown in Fig.3.34, is used to maintain the speed of driven shaft at the same speed as the driving shaft at every instant. To achieve this, the driving and the driven shafts should make equal angles with the intermediate shaft and the forks of the intermediate shaft should lie in the same plane. Let γ be the angle turned by the intermediate shaft while the angle turned by the driving shaft and the driven shaft be θ and ϕ respectively.

Then $$\tan\theta = \cos\alpha \tan\gamma$$

and $$\tan\phi = \cos\alpha\tan\gamma$$

$\therefore$ $$\theta = \phi$$

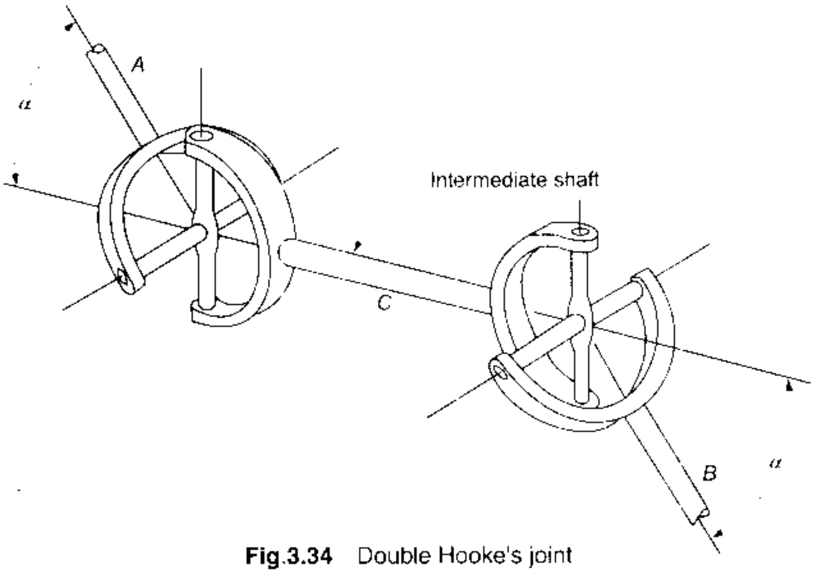


Fig.3.34 Double Hooke's joint

However, if the forks of the intermediate shafts lie in perpendicular planes to each other, then there will be a variation of the speed of the driven shaft.

$$\left(\frac{\omega_C}{\omega_A}\right)_{min} = \cos\alpha$$

$$\left(\frac{\omega_B}{\omega_C}\right)_{min} = \cos\alpha$$

$$\therefore \qquad \left(\frac{\omega_B}{\omega_A}\right)_{min} = \cos^2 \alpha \tag{3.28}$$

Similarly,
$$\left(\frac{\omega_B}{\omega_A}\right)_{max} = \frac{1}{\cos^2 \alpha} \tag{3.29}$$

Example 3.5

The angle between the axes of two shafts connected by a Hooke's joint is 20°. Determine the angle turned through by the driving shaft when the velocity ratio is unity.

■ Solution

For velocity ratio to be unity,

$$\cos\theta = \pm\left[\frac{1}{1+\cos\alpha}\right]^{0.5}$$

$$= \pm\left[\frac{1}{1+\cos 20°}\right]^{0.5} = \pm 0.71801$$

$$\theta = 44.12° \quad \text{or} \quad 135.89°$$

Example 3.6

Two shafts are connected by a Hooke joint. The driving shaft revolves uniformly at 600 rpm. The total variation in the speed of the driven shaft is not to exceed ±5% of the mean speed. Find the greatest permissible angle between the centre lines of the shafts.

■ Solution

Total fluctuation in speed of the driven shaft,

$$= 0.10\omega_m = \omega_m\left[\frac{1-\cos^2\alpha}{\cos\alpha}\right]$$

or
$$\cos^2\alpha + 0.10\cos\alpha - 1 = 0$$

$$\cos\alpha = \frac{\left[-0.10 \pm \{(0.10)^2 + 4\}^{0.5}\right]}{2}$$

$$= 0.95125$$

$$\alpha = 17.96°$$

Example 3.7

Two shafts are connected by a Hooke joint. The driving shaft rotates at a uniform speed of 1200 rpm. The angle between the shafts is 15°. Calculate the maximum and minimum speeds of the driven shaft. When is the acceleration of the driven shaft maximum?

■ Solution

Maximum speed,
$$N_{max} = \frac{N}{\cos\alpha} = \frac{1200}{\cos 15°}$$

$$= 1243.3 \text{ rpm}$$

Minimum speed,
$$N_{min} = N\cos\alpha = 1200\cos 15°$$

$$= 1159.1 \text{ rpm}$$

For acceleration to be maximum,

$$\cos 2\theta \approx \frac{2\sin^2\alpha}{2-\sin^2\alpha}$$

$$\approx \frac{2\sin^2 15°}{2-\sin^2 15°}$$

$$= \frac{2\times 0.06698}{2-2\times 0.06698}$$

$$= 0.07179$$

$$\theta = 43.94°,\ 137.06°,\ 223.94° \text{ and } 317.04°$$

Example 3.8

The driving shaft of a Hooke joint runs at a speed of 300 rpm. The angle between the shafts is 20°. The driven shaft with attached masses has a mass of 60 kg at a radius of gyration of 200 mm. If a steady torque of 500 N m resists rotation of the driven shaft, find the torque required at the driving shaft, when angle turned through by the driving shaft is 45°.

■ **Solution**

Moment of inertia of the driven shaft,

$$I = mK^2$$

$$= 60\times(0.2)^2 = 3.4 \text{ kg.m}^2$$

Angular acceleration of the driven shaft

$$\alpha_B = -\omega_A^2\left[\frac{\cos\alpha\sin 2\theta\sin^2\alpha}{(1-\cos^2\theta\sin^2\alpha)^2}\right]$$

$$= -\left(2\pi\times\frac{300}{60}\right)^2\left[\frac{\cos 20°\sin 90°\sin^2 20°}{(1-\cos^2 45°\sin^2 20°)^2}\right]$$

$$= -986.96\left[\frac{0.10992}{0.88644}\right]$$

$$= -123.38 \text{ rad/s}^2$$

Torque required to accelerate the driven shaft

$$= I\alpha_B$$

$$= -3.4\times 123.38$$

$$= -293.7 \text{ Nm}$$

Total torque required on the driven shaft,

$$T_B = 500-293.7$$

$$= 206.3 \text{ Nm}$$

Torque required on the driving shaft,

$$T_A = T_B\frac{\omega_B}{\omega_A}$$

$$= T_B\left[\frac{\cos\alpha}{(1-\cos^2\theta\sin^2\alpha)}\right]$$

$$= 206.3\left[\frac{\cos 20°}{(1-\cos^2 45°\sin^2 20°)}\right]$$

$$= 205.9 \text{ Nm}$$

Example 3.9

In a double Hooke's coupling, the driving and the driven shafts are parallel and the angle between each and the intermediate shaft is 25°. Find the maximum and minimum velocities of the driven shaft if the axis of the driving pin carried by the intermediate shaft has inadvertently been placed 90° in advance of the correct position. The driving shaft rotates uniformly at 300 rpm.

■ **Solution**

$$(N_B)_{max} = \frac{N_A}{\cos^2\alpha} = \frac{300}{\cos^2 25°}$$
$$= 365.23 \text{ rpm}$$
$$(N_B)_{min} = N_A \cos^2\alpha = 300\cos^2 25°$$
$$= 264.42 \text{ rpm}$$

Exercises

1 Describe the working and applications of a Geneva mechanism.

2 What are the uses of a pantograph? Describe the principle and working of a pantograph.

3 Describe Hart's mechanism with a neat sketch Prove that the tracing point describes a straight line.

4 Name the mechanisms to generate (a) an exact straight line, and (b) an approximate straight line.

5 Describe the working of the following mechanisms:
(a) Geneva wheel, (b) Pawl and ratchet, and (c) lazy tongs.

6 A circle with *AD* as diameter, has a point *B* on its circumference. On *AB* produced there is a point *C* such that if *B* turns about *A*, the product *AB* × *AC* is constant. Prove that the point *C* moves in a straight line perpendicular to *AD* produced.

7 A torque of 100 N m is applied to the link *OC* of a Grasshopper mechanism shown in Fig.3.35. Link *OC* makes an angle of 20° with the horizontal. Find the magnitude of the vertical force exerted at *Q* to overcome this torque. *OC* = 80 mm, *PC* = 120 mm, and *PQ* = 300 mm. Also calculate the force required if the link makes an angle of 0°.

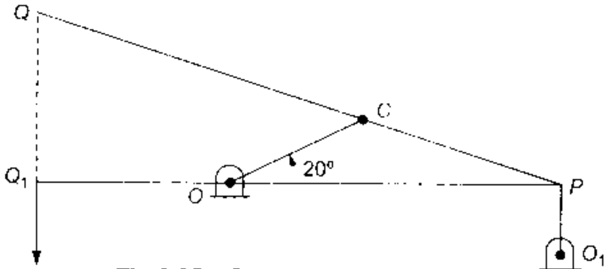


Fig.3.35 Grasshopper mechanism

8 The distance between the fixed centres of a Watt straight line mechanism shown in Fig.3.36 is 320 mm. The lengths of links are *OA* = 300 mm, *AB* = 400 mm, and *BO*$_1$ = 250 mm. Locate the position of a point *P* on *AB* which will trace approximately straight line.

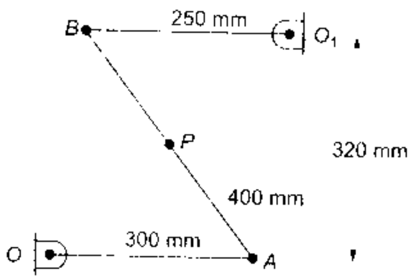


Fig.3.36 Watt mechanism

9 (a) Describe the straight line mechanisms.

(b) Sketch and describe any mechanism which can give an exact generated straight line motion. Give the mathematical analysis involved.

(c) Describe the Pantograph with a neat sketch and state its use.

10 In the Robert mechanism shown in Fig.3.37, if $AB = BC = CD = AD/2$, locate the point P on the central vertical arm that approximately describes a straight line.

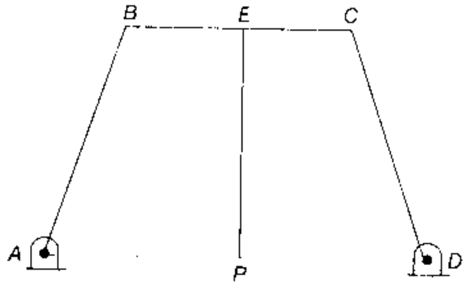


Fig.3.37 Robert mechanism

11 In the Watt mechanism shown in Fig.3.38, plot the path of point P and mark and measure the straight line segment of the path of P.

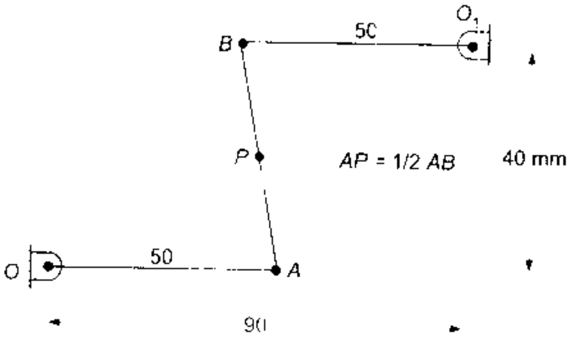


Fig.3.38 Watt mechanism

12 The distance between the axes of a car using Davis steering gear is 3.3 m. The steering pivots are 1.3 m apart. While moving in a straight line, determine the inclination of the track arms to the longitudinal axis of the car.

13 A car using Ackermann steering gear has a wheel base of 3.8 m and a track of 1.5 m. The track rod is 1.2 m and each track arm is 150 mm long. The distance between the pivots of front stub axles is 1.25 m. If the car is turning to the right find the radius of curvature of the path followed by the inner front wheel for correct steering.

14 The distance between the pivots of the front stub axles of a car is 1.35 m. The length of track rod is 1.25 m. The wheel track is 1.5 m and the wheel base is 3.8 m. What should be the length of the track arm if the gear is to be given a correct steering, when rounding a corner of radius 5 m.

15 A Hooke coupling is used to connect two shafts whose axes are inclined at 30°. The driving shaft rotates uniformly at 600 rpm. What are the extreme angular velocities of the driven shaft? Find the maximum value of retardation or acceleration and state the angle where both will occur.

16 Two shafts are to be joined by a Hooke coupling. The driving shaft rotates at a uniform speed of 600 rpm and the speed of the driven shaft must lie between 500 and 550 rpm. Determine the maximum permissible angle between the shafts.

17 A Hooke coupling connects a shaft running at a uniform speed of 900 rpm to a second shaft. The angle between their axes being 20°. Find the velocity and acceleration of the driven shaft at the instant when the fork of the driving shaft has turned through an angle of 15° from the plane containing the shaft axes.

18 The angle between the axes of two horizontal shafts to be connected by a Hooke joint is 150°. The speed of the driving shaft is 150 rpm. The driven shaft carries a flywheel of mass 15 kg and has a radius of gyration of 100 mm. If the forked end of the driving shaft rotates 30° from the horizontal plane, find the torque required to drive the shaft to overcome the inertia of the flywheel.

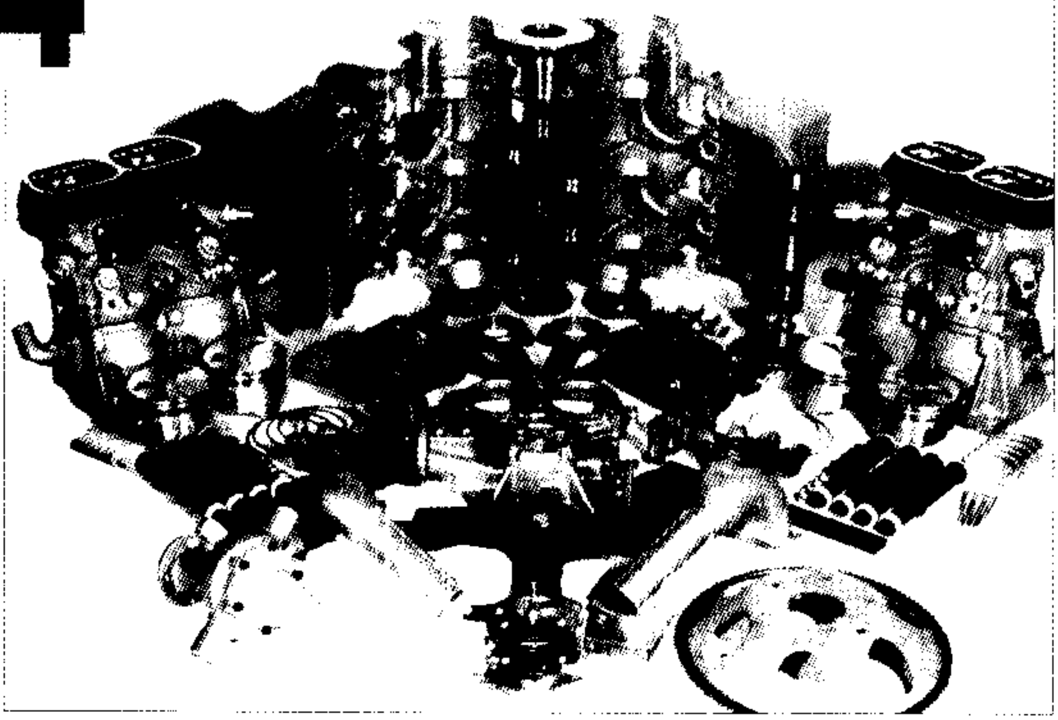
19 The driving shaft of a double Hooke's joint rotates at 500 rpm. The angle of the driving and driven shafts with the intermediate shaft is 25°. Determine the maximum and minimum velocities of the driven shaft.

20 (a) State precisely what an automobile steering is expected to achieve. Establish the relationship among the parameters of the mechanism necessary for this purpose. State the basic difference between the Ackermann and Davis steering mechanisms and compare them in respect to their relative usefulness in automobiles and their relative faithfulness to the ideal objective.

(b) What is a Hooke joint? Explain what it is expected to achieve, and how it does so with the help of kinematic diagram. State a few applications of this joint and comment on the variation of the velocity ratio between two parallel shafts connected through a spindle having Hooke's joints at both its ends.

21 In a Hooke joint, prove that if the angle between shafts is small, the total fluctuation of velocity ratio varies as the square of the shaft angle.

4



FRICTION

4.1 INTRODUCTION

When a body moves or tends to move on another body, the property of the two bodies by virtue of which a force is developed between the two bodies, which opposes the motion, is called *friction*. This force is called the *force of friction*. The force of friction acts opposite to the impending motion.

Consider a block of weight W resting on a plane rough surface, as shown in Fig.4.1. The normal reaction due to the weight of the block is R. Let a force P be applied to the block towards the right. A frictional force F will be set up between the block and the rough surface. The direction of F will be towards the left. The *coefficient of friction* (μ) is the ratio of the force of friction (F) to the normal reaction (R). Thus

$$\mu = \frac{F}{R} \tag{4.1}$$

Let S be the resultant of F and R making an angle of ϕ with R. Then ϕ is called the *angle of friction*, such that

$$\tan \phi = \frac{F}{R} \tag{4.2}$$

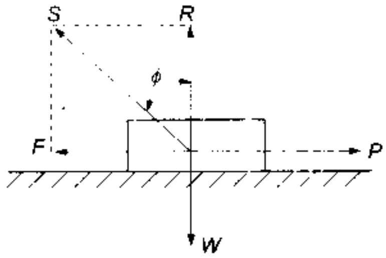


Fig.4.1 Body resting on rough horizontal plane

From (4.1) and (4.2), we find that

$$\mu = \tan\phi \tag{4.3}$$

Therefore, in the case of limiting friction, the coefficient of friction is equal to the tangent of the angle of friction.

Now consider the block resting on a rough inclined plane, as shown in Fig.4.2. Resolving the forces along and perpendicular to the plane, we have

$$F = W\sin\theta$$
$$R = W\cos\theta$$

or

$$\frac{F}{R} = \tan\theta$$

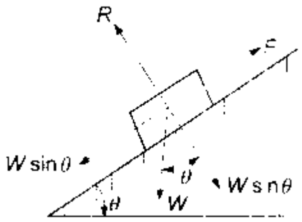


Fig.4.2 Body resting on rough inclined plane

Comparing with (4.2), we find that

$$\tan\theta = \tan\phi$$

or

$$\theta = \phi \tag{4.4}$$

Thus, in the case of limiting friction, the angle of the plane is equal to the angle of friction. The angle of the plane when motion of an object on the plane is impending is called the *angle of repose*. This is the maximum angle that a heap of sand or similar materials will make with the horizontal.

If the force P is made to revolve about a vertical axis, the resultant S will also revolve about a vertical axis. As S revolves, it will generate a cone of vertex angle 2ϕ. This cone is called the *cone of friction*, as shown in Fig.4.3.

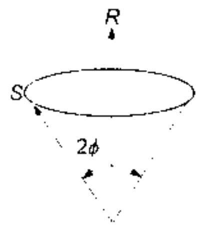


Fig.4.3 Cone of friction

4.2 Types of Friction

Friction is of the following types:

Static Friction This is the friction experienced by two bodies in contact, while at rest.

Dynamic (or Kinetic) Friction This is the friction experienced by two bodies in contact, while in motion. It is less than the static friction.

Sliding Friction This is due to sliding of two bodies on each other.

Rolling Friction This is due to rolling of two bodies on each other.

Pivot Friction This is the friction experienced by two bodies due to motion of rotation of one body in the other, as in the case of a foot step bearing.

Dry (or Solid) Friction This is due to two dry and unlubricated surfaces in contact.

Boundary (or Skin) Friction This is the friction experienced by two bodies separated by a very thin layer of lubricant.

Fluid (or Film) Friction This is the friction experienced by two bodies in contact when separated by a thick film of lubricant.

4.3 LAWS OF FRICTION

Static friction

1. The force of friction always acts in a direction opposite to the impending motion.
2. The limiting force of friction is proportional to the normal reaction.
3. The force of friction is independent of the area of contact between the two surfaces.
4. The force of friction depends upon the roughness of the surfaces of the two materials.

Kinetic friction

1. The force of friction always acts in a direction opposite to that in which the body is moving.
2. The magnitude of kinetic friction bears a constant ratio to the normal reaction between the two surfaces.
3. The force of friction is independent of the relative velocity between the two surfaces in contact, but it decreases slightly with increase in velocity.
4. The force of friction increases with reversal of motion.
5. The coefficient of friction changes slightly when temperature changes.

Fluid friction

1. The force of friction is almost independent of the load.
2. The force of friction reduces with the increase of temperature of the lubricant.
3. The force of friction depends upon the type and viscosity of the lubricant.
4. The force of friction is independent of the nature of surfaces.
5. The frictional force increases with the increase in the relative velocity of the frictional surfaces.

4.4 FORCE ANALYSIS OF A SLIDING BODY

4.4.1 Body Resting on a Horizontal Plane

Consider a body of weight W resting on a rough horizontal plane being pulled by a force P inclined at an angle θ, as shown in Fig.4.4(a). Resolving the forces horizontally and vertically, we have

$$P\cos\theta = F$$

$$R + P\sin\theta = W$$

Now
$$F = \mu R$$

Therefore
$$P\cos\theta = \mu R$$

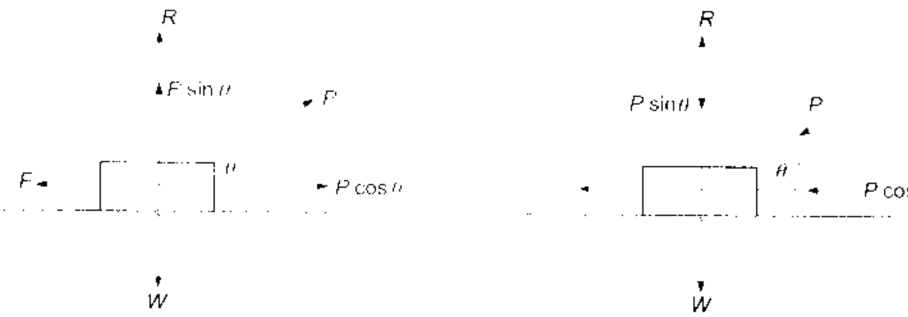
$$\frac{P\cos\theta}{\mu} + P\sin\theta = W$$

$$P\left(\frac{\cos\theta}{\mu} + \sin\theta\right) = W$$

$$P(\cos\theta\cos\phi + \sin\phi\sin\theta) = W\sin\phi$$

or
$$P\cos(\theta - \phi) = W\sin\phi$$

or
$$P = \frac{W\sin\phi}{\cos(\theta - \phi)} \qquad (4.5)$$



(a) Pull type force (b) Push type force

Fig.4.4 Body resting on rought horizontal plane

For P to be minimum, $\cos(\theta - \phi)$ should be maximum, that is,

$$\cos(\theta - \phi) = 1$$

or
$$\theta = \phi$$

and
$$P_{min} = W\sin\theta \qquad (4.6)$$

If the force P is of the push type, as shown in Fig.4.4(b), then

$$P\cos\theta = F = \mu R$$
$$P\sin\theta + W = R = \frac{P\cos\theta}{\mu}$$
$$P(\cos\theta\cos\phi - \sin\theta\sin\phi) = W\sin\phi$$
$$P\cos(\theta+\phi) = W\sin\phi$$

or
$$P = \frac{W\sin\phi}{\cos(\theta+\phi)} \tag{4.7}$$

4.4.2 Body Resting on an Inclined Plane with Force Inclined to the Plane

1. **Body going up the plane**

Consider the body resting on the inclined plane as shown in Fig.4.5(a). Resolving the forces perpendicular and along the plane, we have

$$P\cos\theta = W\sin\alpha + \mu R$$
$$W\cos\alpha = P\sin + R$$

or $$P\cos\theta = W\sin\alpha - \tan\phi(W\cos\alpha \quad P\sin\theta)$$

or $$P(\cos\theta - \sin\theta\tan\phi) = W(\sin\alpha + \cos\alpha\tan\phi)$$

or $$P(\cos\theta\cos\phi - \sin\theta\sin\phi) = W(\sin\alpha\cos\phi + \cos\alpha\sin\phi)$$

$$P\cos(\theta-\phi) = W\sin(\alpha-\phi)$$
$$P = W\sin(\alpha-\phi)/\cos(\theta-\phi) \tag{4.8}$$

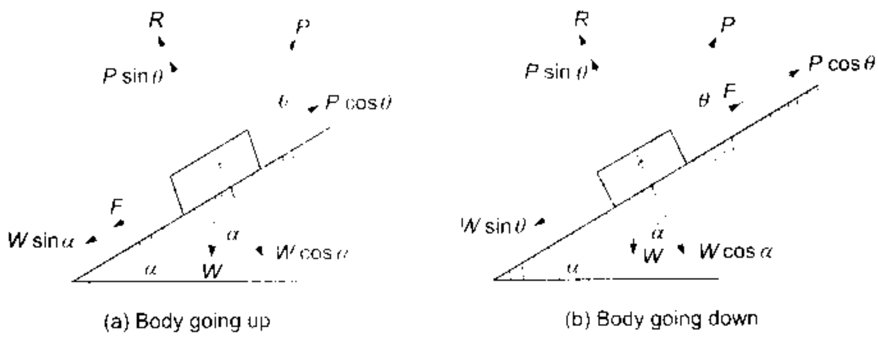


Fig.4.5 Body resting on inclined plane

For P to be minimum, $\cos(\theta-\phi) = 1$ or $\theta = \phi$. In that case,

$$P_{min} = W\sin(\alpha+\phi) \tag{4.9}$$

Body going down the plane

When the body is going down the plane, as shown in Fig.4.5(b), then we get

$$P = \frac{W\sin(\phi-\alpha)}{\cos(\theta+\phi)} \tag{4.10}$$

For P to be minimum, $\cos(\theta+\phi) = $, or $\phi = -\theta$. In that case

$$P_{min} = W\sin(\phi-\alpha) \tag{4.11}$$

4.4.3 Body Resting on an Inclined Plane with Horizontal Force

1. **Body going up the plane**

Consider the body as shown in Fig.4.5(a). Resolving the forces perpendicular and along the plane, we have

$$P\cos\alpha = W\sin\alpha + \mu R$$

$$P\sin\alpha + W\cos\alpha = R$$

or $$P\cos\alpha = W\sin\alpha + \tan\phi(P\sin\theta + W\cos\alpha)$$

or $$P(\cos\alpha\cos\phi - \sin\alpha\sin\phi) = W(\sin\alpha\cos\phi + \cos\alpha\sin\phi)$$

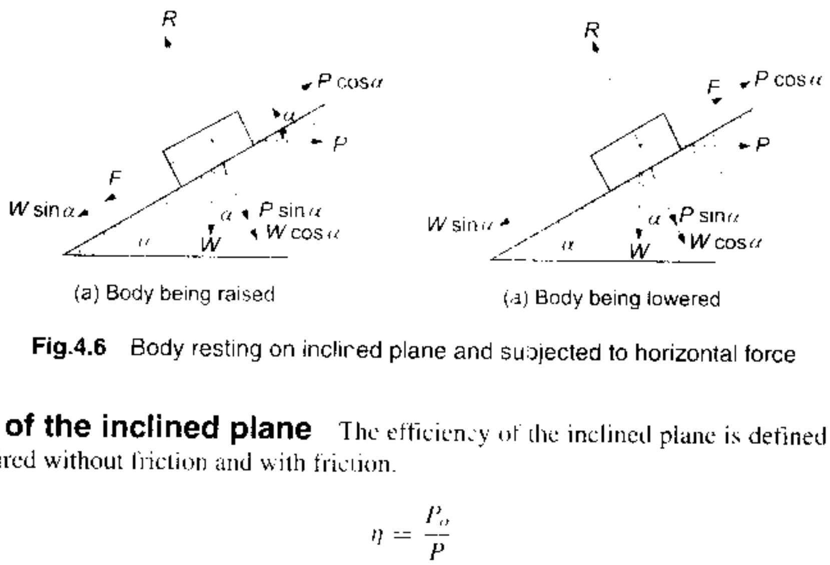
or $$P\cos(\alpha + \phi) = W\sin(\alpha + \phi)$$

$$P = W\tan(\alpha + \phi) \tag{4.12}$$

2. **Body going down the plane**

For the body shown in Fig.4.6(b), we get

$$P = W\tan(\varphi - \alpha) \tag{4.13}$$



(a) Body being raised

(a) Body being lowered

Fig.4.6 Body resting on inclined plane and subjected to horizontal force

Efficiency of the inclined plane The efficiency of the inclined plane is defined as the ratio of the effort required without friction and with friction.

$$\eta = \frac{P_o}{P}$$

1. For body going up the plane with inclined force

$$\eta = \frac{\frac{\sin\alpha}{\cos\theta}}{\sin(\alpha + \phi)/\cos(\theta - \phi)} = \frac{1 - \mu\tan\theta}{1 + \mu\cot\alpha} \tag{4.14}$$

2. For body going down the plane with inclined force

$$\eta = \left(\frac{\sin\alpha}{\cos\theta}\right)\cdot\left[\frac{\cos(\theta + \phi)}{\sin(\phi - \alpha)}\right]$$

$$= \frac{1 - \mu\tan\theta}{1 - \mu\cot\alpha} \tag{4.15}$$

3. For body going up the plane with horizontal force

$$\eta = \frac{\tan\alpha}{\tan(\alpha + \phi)} \tag{4.16}$$

4. For body going down the plane with horizontal force

$$\eta = \frac{\tan(\alpha - \phi)}{\tan\alpha} \tag{4.17}$$

4.5 SCREW FRICTION

Screws are used for fastening, load lifting and power transmission purposes. Screws may have single start or multi-start threads. Lead is the product of pitch and number of starts.

Let p = pitch of the threads

L = lead of the threads

d = mean pitch circle diameter of the screw

α = helix angle, that is the inclination of the threads with the horizontal.

Then

$$\tan\alpha = \frac{p}{\pi d} \quad \text{or} \quad \frac{L}{\pi d} \tag{4.18}$$

4.5.1 Screw Jack

A screw jack is a device for lifting loads. The principle of working of a screw jack is similar to that of an inclined plane. Figure 4.7 shows the common form of a screw jack.

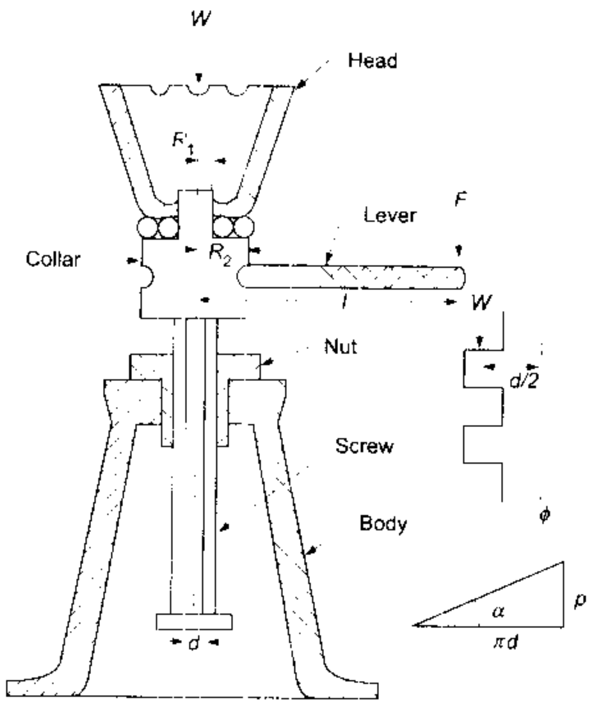


Fig.4.7 Screw jack

Let P = effort applied at the circumference of the screw to lift the load

W = load to be lifted

μ = coefficient of friction between the screw and the nut

Raising the load As derived in Section 4.4, for raising the load, we have

$$P = W\tan(\alpha + \phi)$$

Torque required to overcome friction between the screw and nut,

$$T_1 = P \times d/2 = Wd\tan(\alpha + \phi)/2$$

Torque required to overcome friction at the collar,

$$T_2 = \mu_c W d_m/2$$

where μ_c = collar coefficient of friction

d_m = mean diameter of the collar

Total torque, $$T = T_1 + T_2 = (Pd + \mu_c W d_m)/2 \qquad (4.19)$$

Let F = be the effort applied at the end of a lever of length l.

Then $$T = Fl \qquad (4.20)$$

Lowering the load For lowering of load, we have

$$P = W\tan(\phi - \alpha)$$

The rest of the formulae remain the same as for raising the load.

Efficiency of the screw jack

Efficiency, $$\eta = \frac{\tan\alpha}{\tan(\alpha + \phi)}, \text{ for raising the load}$$

$$= \frac{\tan(\phi - \alpha)}{\tan\alpha}, \text{ for lowering the load}$$

For efficiency to be maximum, $\frac{d\eta}{d\alpha} = 0$.

Now
$$\eta = \frac{\tan\alpha(1 - \tan\alpha\tan\phi)}{\tan\alpha + \tan\phi}$$

$$= \left(\frac{\sin\alpha}{\cos\alpha}\right)\frac{\left[1 - \left(\frac{\sin\alpha}{\cos\alpha}\right)\left(\frac{\sin\phi}{\cos\phi}\right)\right]}{\left[\frac{\sin\alpha}{\cos\alpha} + \frac{\sin\phi}{\cos\phi}\right]}$$

$$= \frac{\sin\alpha\,[\cos\alpha\cos\phi - \sin\alpha\sin\phi]}{\cos\alpha[\sin\alpha\cos\phi + \sin\phi\cos\alpha]}$$

$$= \frac{\sin\alpha\cos(\alpha + \phi)}{\cos\alpha\sin(\alpha + \phi)}$$

$$= \frac{\sin(2\alpha + \phi) - \sin\phi}{\sin(2\alpha + \phi) + \sin\phi}$$

For efficiency to be maximum, $\sin(2\alpha + \phi) = 0$.

$$\text{or} \qquad 2\alpha + \phi = 90°$$

$$\text{or} \qquad \alpha = \frac{45° - \phi}{2}$$

$$\eta_{max} = \frac{1 - \sin\phi}{1 + \sin\phi} \qquad (4.21)$$

Self-locking and overhauling screw The effort required for lowering the load is,

$$P = W\tan(\phi - \alpha)$$

Torque,

$$T = P\left(\frac{d}{2}\right) = W\left(\frac{d}{2}\right)\tan(\phi - \alpha)$$

Torque will remain positive if $\phi > \alpha$. Such a screw is called a *self-locking screw*. If $\phi < \alpha$, then the torque will become negative. In other words, the screw will lower on its own. Such a screw is called an *over hauling* screw.

For $\alpha = \phi$, the efficiency of the screw becomes,

$$\eta = \frac{\tan\phi}{\tan 2\phi}$$

For a self locking screw,

$$\eta \le \frac{\tan\phi}{\tan 2\phi} \le \left[1/2 - \tan^2\phi/2\right] \le 50\%$$

Screw having V-threads For a screw having V-threads, as shown in Fig.4.8, let 2β be the angle of the threads.

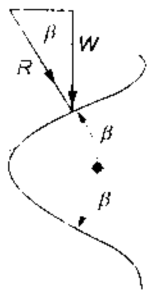


Fig.4.8 V-threads

Then, the normal reaction on the threads,

$$R = \frac{W}{\cos\beta}$$

Frictional force,

$$F = \mu R = \left(\frac{\mu}{\cos\beta}\right)\cdot W$$

$$= \mu_e W$$

where μ_e is called the virtual or equivalent coefficient of friction.

Therefore, for a screw having V-threads, the virtual coefficient of friction should be used to calculate the torque required to lift the load.

Example 4.1

A square threaded bolt of root diameter 20 mm and pitch 5 mm is tightened by screwing a nut whose mean diameter of bearing surface is 50 mm. The coefficient of friction for nut and bolt is 0.10 and for nut and bearing surface is 0.15. Find the force required at the end of a spanner 450 mm long when the load on the bolt is 10 kN.

■ **Solution**

$$\text{Mean diameter of screw, } d_m = d_c + \frac{p}{2} = 20 + 2.5 = 22.5 \text{ mm}$$

$$\tan\alpha = \frac{p}{\pi d_m} = \frac{5}{(\pi \times 22.5)} = 0.07073$$

$$\alpha = 4.05^\circ$$

$$\phi = \tan^{-1} 0.10 = 5.71^\circ$$

$$P = W \tan(\alpha - \phi)$$

$$= 10 \times 10^3 \times \tan(4.05 + 5.71)^\circ = 1720.1 \text{ N}$$

$$T = \frac{Pd_m}{2} + \mu_c W R_m$$

$$= 1720.1 \times \frac{22.5}{2} + 0.15 \times 10^4 \times 25$$

$$= 19351.12 + 37500 = 56851.12 \text{ Nmm}$$

$$Fl = T$$

$$F \times 450 = 56851.12$$

$$F = 126.33 \text{ N}$$

Example 4.2

The mean diameter of a bolt having V-threads is 25 mm. The pitch of the thread is 5 mm and the angle of threads is 55°. The bolt is tightened by screwing a nut whose mean radius of bearing surface is 25 mm. The coefficient of friction for nut and bolt is 0.10 and for nut and bearing is 0.15. Find the force required at the end of a lever 0.5 m long when the load on the bolt is 15 kN.

■ **Solution**

$$\alpha = \tan^{-1}\left(\frac{5}{25\pi}\right) = 3.64^\circ$$

$$\phi = \tan^{-1}\left(\frac{0.1}{\cos 27.5^\circ}\right) = 6.43^\circ$$

$$P = W \tan(\alpha + \phi)$$

$$= 15 \times 10^3 \times \tan(3.64 + 6.43)^\circ = 2663.8 \text{ N}$$

$$T = \frac{Pd_m}{2} + \mu_c W R_m$$

$$= 2663.8 \times 12.5 + 0.15 \times 15000 \times 25$$

$$= 33297.5 + 56250 = 89547.5 \text{ N}$$

$$Fl = T$$

$$F = \frac{89547.5}{500} = 179.1 \text{ N}$$

Example 4.3

Two tie rods are connected by a turnbuckle having right and left hand metric threads of V-type. The pitch of the threads is 5 mm on a mean diameter of 30 mm and a thread angle of 60°. Assuming coefficient of friction of 0.12, find the torque required to produce a pull of 40 kN.

■ **Solution**

$$\mu_e = \frac{\mu}{\cos\beta} = \frac{0.12}{\cos 30°} = 0.13856$$

$$\phi = \tan^{-1}\mu_e = 7.89°$$

$$\alpha = \tan^{-1}(p/\pi d) = \frac{5}{(\pi \times 30)} = 3.04°$$

(a) When rods are tightened

$$P = W\tan(\alpha + \phi)$$
$$= 40 \times 10^3 \times \tan(3.04 + 7.89)°$$
$$= 7724.5 \text{ N}$$

Torque, $$T = \frac{Pd}{2}$$
$$= 7724.5 \times 15 \times 10^{-3} = 115.86 \text{ Nm}$$

(b) When the rods are slackened

$$P = W\tan(\phi - \alpha)$$
$$= 40 \times 10^3 \times \tan(7.89 - 3.04)$$
$$= 3394 \text{ N}$$
$$T = 3394 \times 15 \times 10^{-3}$$
$$= 50.91 \text{ Nm}$$

4.6 FLAT PIVOT BEARING

A flat pivot bearing like the foot step bearing is shown in Fig.4.9.

Let W = load on the bearing
R = radius of bearing surface
p = intensity of pressure between rubbing surfaces

4.6.1 Uniform Pressure

When the pressure is uniformly distributed over the bearing surface area, then

$$p = \frac{W}{\pi R^2}$$

Consider a ring of radius r and thickness d· of the bearing area, as shown in the figure.

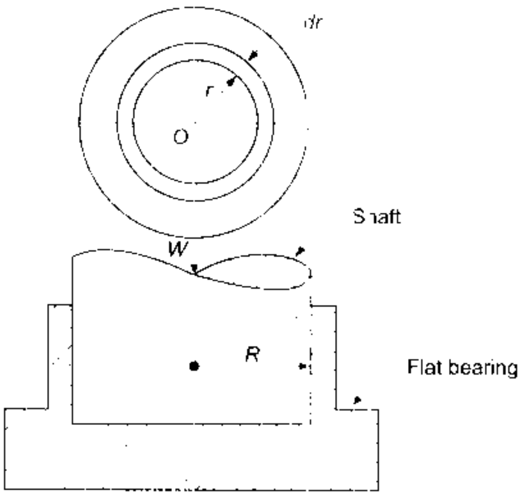


Fig.4.9 Flat pivot bearing

Area of bearing surface, $\delta A = 2\pi r \cdot dr$

Load transmitted to the ring, $\delta W = p \cdot \delta A$

Frictional resistance to sliding at radius r,

$$F_r = \mu \cdot \delta W = 2\pi\mu \cdot pr\, dr$$

Frictional torque on the ring,
$$dT_f = F_r r = 2\pi\mu \cdot pr^2\, dr$$

Total frictional torque,
$$T_f = \int_0^R 2\pi\mu p r^2\, dr = 2\pi\mu p \int_0^R r^2\, dr$$

$$= 2\pi\mu p \frac{R^3}{3}$$

$$= 2\mu W \frac{R}{3} \quad (4.22)$$

Power lost in friction,
$$P = T_f \cdot \omega$$

where $\omega = \frac{2\pi N}{60}$ rad/s and N is the speed of the shaft in rpm.

If the shaft of radius R_2 is resting on a disc of radius R_1, then

$$T_f = 2\pi\mu p \int_{R_1}^{R} r^2 dr$$

$$= 2\pi\mu p \frac{(R_2^3 - R_1^3)}{3} \quad (4.23)$$

and
$$p = \frac{W}{\pi\left(R_2^2 - R_1^2\right)}$$

4.6.2 Uniform Wear

The rate of wear depends upon the intensity of pressure and the rubbing velocity. The rubbing velocity increases with the increase in radius. Therefore, the wear rate is proportional to the product of pressure and radius. For uniform wear,

$$pr = C$$

or

$$p = \frac{C}{r}$$

where C is a constant.

Load transmitted to the ring,

$$\delta W = p \cdot 2\pi r \, dr = 2\pi C \, dr$$

Total load transmitted to the bearing,

$$W = 2\pi C \int_0^R dr = 2\pi C R$$

or

$$C = \frac{W}{2\pi R}$$

Frictional torque acting on the ring,

$$dT_f = 2\pi \mu p r^2 \, dr = 2\pi \mu C r \, dr$$

Total frictional torque on the bearing,

$$T_f = 2\pi \mu C \int_0^R r \, dr = \pi \mu C R^2 = \mu W \frac{R}{2} \tag{4.24}$$

If the shaft of radius R_2 is resting on a disc of inner radius R_1, then

$$T_f = 2\pi \mu C \int_{R_1}^{R_2} r dr = \pi \mu C \left(R_2^2 - R_1^2\right) = \mu W \frac{(R_2 + R_1)}{2} = \mu W R_m \tag{4.25}$$

4.7 CONICAL PIVOT BEARING

Consider a conical pivot bearing as shown in Fig.4 10.

Let p_n = intensity of normal pressure on the cone;

α = semicone angle;

μ = coefficient of friction between the shaft and the bearing and

R = radius of the shaft.

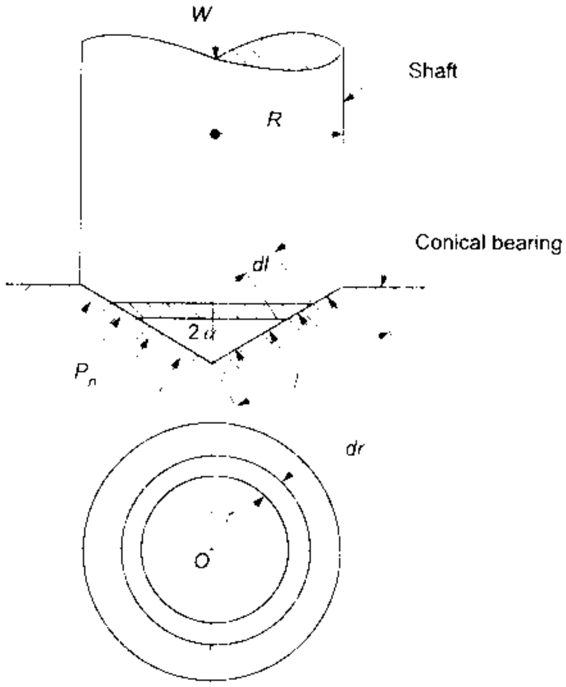


Fig.4.10 Conical pivot bearing

Consider a small ring of radius r and thickness dr. Let dl be the length of the ring along the cone, so that

$$dl = dr \cdot \text{cosec}\,\alpha$$

Area of the ring, $$dA = 2\pi r \cdot dl = 2\pi r \cdot dr\,\text{cosec}\,\alpha$$

4.7.1 Uniform Pressure

Normal load acting on the ring, $$\delta W_n = 2\pi r \cdot p_n \cdot dr\,\text{cosec}\,\alpha$$

Vertical load acting on the ring, $$\delta W = \delta W_n \sin\alpha$$

Total vertical load transmitted to the bearing, $$W = \int_0^R \delta W = 2\pi p_n \int_0^R r\,dr$$

$$= \pi R^2 p_n$$

or $$p_n = \frac{W}{\pi R^2}$$

Frictional force acting on the ring tangentially at radius r, $F_f = \mu \cdot \delta W_n$

Frictional torque acting on the ring $$dT_f = F_f \cdot r$$

$$= 2\pi\mu \cdot p_n\,\text{cosec}\,\alpha \cdot r^2 \cdot dr$$

Total frictional torque, $$T_f = 2\pi\mu \cdot p_n \cdot \text{cosec}\,\alpha \int_0^R r^2\,dr$$

$$= \left(\frac{2}{3}\right) \cdot \pi R^3 \cdot \mu p_n \cdot \text{cosec}\,\alpha$$

$$= \left(\frac{2}{3}\right) \cdot \mu W R\,\text{cosec}\,\alpha \tag{4.26}$$

4.7.2 Uniform Wear

In the case of uniform wear, the intensity of normal pressure varies inversely with the distance. Therefore

$$p_n \cdot r = C, \quad \text{where } C \text{ is a constant}$$

Load transmitted to the ring, $\delta W = p_n \cdot 2\pi r \cdot dr = 2\pi C \cdot dr$

Total load transmitted to the bearing, $W = \int_0^R \delta W = 2\pi C \int_0^R dr = 2\pi C \cdot R$

or $C = \dfrac{W}{2\pi R}$

Frictional torque acting on the ring, $dT_f = 2\pi\mu \cdot p_n \cdot \text{cosec}\,\alpha \cdot r^2 \cdot dr$

$$= 2\pi\mu \cdot C \cdot \text{cosec}\,\alpha \cdot r \cdot dr$$

Total frictional torque acting on the bearing, $T_f = 2\pi\mu \cdot C \cdot \text{cosec}\,\alpha \int_0^R r \cdot dr$

$$= \pi\mu \cdot C \cdot R^2 \cdot \text{cosec}\,\alpha$$

$$= \frac{1}{2} \times \mu W R\text{cosec}\,\alpha \tag{4.27}$$

4.8 TRUNCATED CONICAL PIVOT BEARING

Consider a truncated conical pivot bearing as shown in Fig.4.11. Then

Intensity of uniform pressure, $p_n = \dfrac{W}{\pi}\left(r_2^2 - r_1^2\right)$

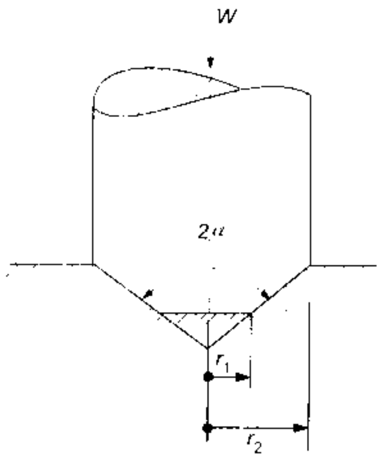


Fig.4.11 Trapezoidal pivot bearing

4.8.1 Uniform Pressure

Total frictional torque acting on the bearing,

$$T_f = 2\pi\mu p_n \operatorname{cosec}\alpha \int_{r_1}^{r_2} r^2 \, dr$$

$$= 2\pi\mu p_n \frac{\operatorname{cosec}\alpha \left(r_2^3 - r_1^3\right)}{3}$$

$$= 2\pi\mu \left[\frac{W}{\pi\left(r_2^2 - r_1^2\right)}\right] \frac{\operatorname{cosec}\alpha \left(r_2^3 - r_1^3\right)}{3}$$

$$= \left(\frac{2}{3}\right) \mu W \cdot \operatorname{cosec}\alpha \left[\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right] \quad (4.28)$$

4.8.2 Uniform Wear

Total frictional torque acting on the bearing,

$$T_f = 2\pi\mu C \operatorname{cosec}\alpha \int_{r_1}^{r_2} r \, dr$$

$$= \pi\mu C \operatorname{cosec}\alpha \left(r_2^2 - r_1^2\right)$$

$$= \pi\mu \left[\frac{W}{2\pi\left(r_2 - r_1\right)}\right] \operatorname{cosec}\alpha \left(r_2^2 - r_1^2\right)$$

$$= \mu W \operatorname{cosec}\alpha \frac{\left(r_2 + r_1\right)}{2}$$

$$= \mu W r_m \operatorname{cosec}\alpha \quad (4.29)$$

4.9 FLAT COLLAR BEARING

A single collar bearing is shown in Fig.4.12(a) and the multiple collar bearing in Fig.4.12(b). Let

r_1, r_2 = inner and outer radii of the bearing respectively

Area of the bearing surface, $A = \pi\left(r_2^2 - r_1^2\right)$

4.9.1 Uniform Pressure

Intensity of pressure, $p = W/A = \dfrac{W}{\pi\left(r_2^2 - r_1^2\right)}$

Frictional torque on the ring of radius r and thickness dr,

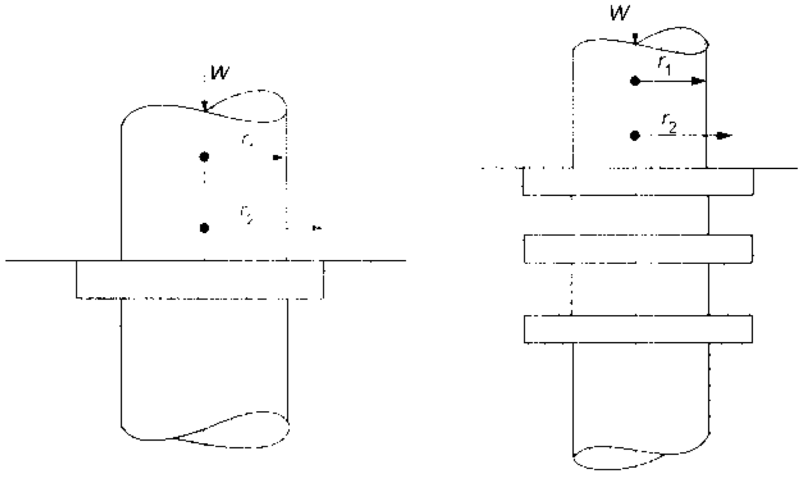
$$dT_f = 2\pi\mu \cdot p \cdot r^2 \cdot dr$$

Total frictional torque,

$$T_f = 2\pi\mu p \int_{r_1}^{r_2} r^2 \cdot dr$$

$$= \left(\frac{2}{3}\right) \cdot \pi\mu p \left(r_2^3 - r_1^3\right)$$

$$= \left(\frac{2}{3}\right) \cdot \mu W \left[\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right] \quad (4.30)$$



(a) Single collar bearing (b) Multiple collar bearing

Fig.4.12 Collar bearings

4.9.2 Uniform Wear

For uniform wear, the load transmitted on the ring,

$$\delta W = p_n \cdot 2\pi r \cdot dr = 2\pi C \cdot dr$$

Total load transmitted to the collar,

$$W = 2\pi C \int_{r_1}^{r_2} dr = 2\pi C (r_2 - r_1)$$

or

$$C = \frac{W}{2\pi (r_2 - r_1)}$$

Frictional torque on the ring,

$$dT_f = \mu \cdot \delta W \cdot r = 2\pi \cdot \mu \cdot C \cdot r \cdot dr$$

Total frictional torque on the bearing,

$$T_f = 2\pi\mu \cdot C \int_{r_1}^{r_2} r \cdot dr = \pi\mu \cdot C \left(r_2^2 - r_1^2\right) = \mu W \frac{(r_1 + r_2)}{2} = \mu W r_m \qquad (4.31)$$

4.10 ROLLING FRICTION

Consider a cylinder or sphere rolling on a flat surface, as shown in Fig.4.13. When there is no deformation of the surface on which rolling is taking place, then the point of contact will be a line in the case of a cylinder and a point in the case of a sphere, as shown in Fig.4.13(a). If the surface deforms, then the shape of the surface will be as shown in Fig.4.13(b). Let the distance between the point of contact B and the point A through which the load W passes be b and F be the force required for rolling. Then rolling moment is equal to $F \cdot h$ and the resisting moment is $W \cdot b$. For the equilibrium of forces, we have

$$F \cdot h = W \cdot b$$

where b is known as the coefficient of rolling friction, and has linear dimension.

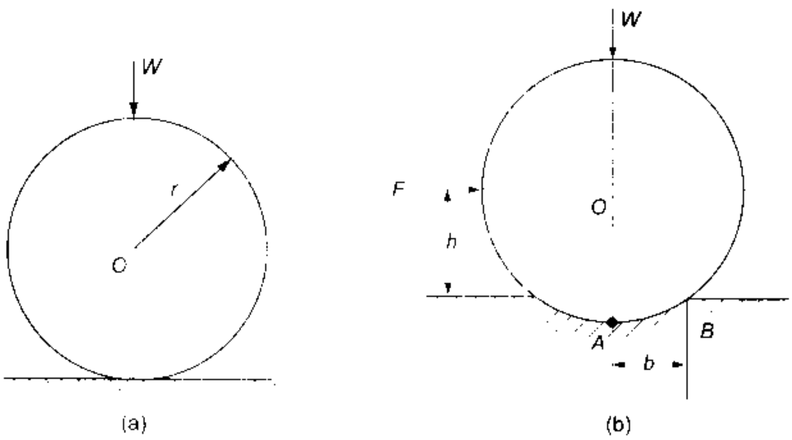


Fig.4.13 Rolling of a cylinder or sphere

Let F_r = force applied to the body for rolling
F_s = force applied to the body for sliding

Then $$F_r = \left(\frac{b}{h}\right) \cdot W$$

and $$F_s = \mu \cdot W$$

The body rolls without sliding, if $F_r < F_s$ or $\mu > \frac{b}{h}$. The body will slide if $F_s < F_r$ or $\mu < \frac{b}{h}$. The body will either roll or slide if $\mu = \frac{b}{h}$.

4.11 FRICTION CIRCLE

Consider a journal bearing, as shown in Fig.4.14. When the journal is at stand still, then the point of contact is at A. The load W is balanced by the reaction R. When the journal starts rotating in the clockwise direction, then the point of contact shifts from A to B. The resultant of normal reaction R and force of friction $F = \mu R$ is S, as shown in Fig.4.14(b). For the equilibrium of the journal, we have

$$W = S$$

Torque,
$$\begin{aligned} T &= W \cdot OC \\ &= W \cdot OB \sin \phi \\ &= W \cdot r \tan \phi \qquad \text{(angle } \phi \text{ being small)} \\ &= W \cdot r\mu \qquad (4.32) \end{aligned}$$

A circle drawn with centre O and radius $OC = r \sin \phi = r\mu$ is called the *friction circle*.

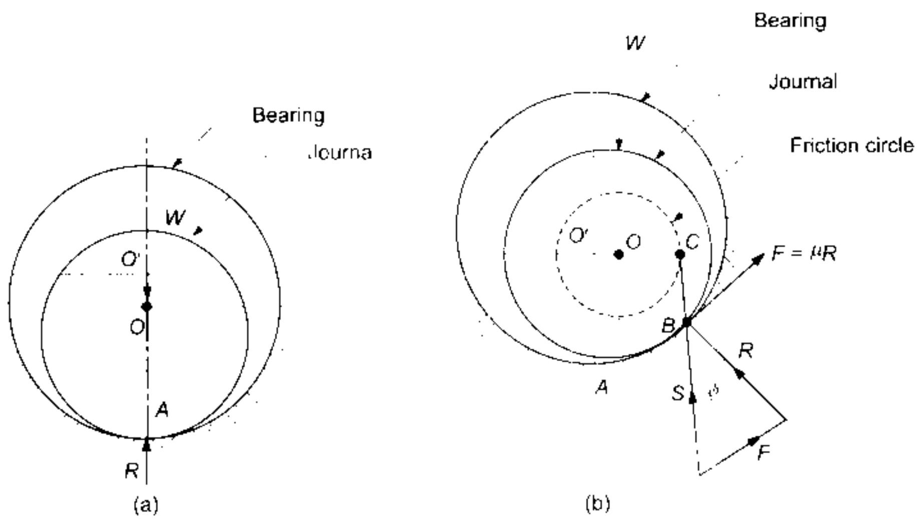


Fig.4.14 Firction circle

Example 4.4

A vertical shaft 140 mm diameter rotating at 120 rpm rests on a flat end footstep bearing. The shaft carries a vertical load of 30 kN. The coefficient of friction is 0.06. Estimate the power lost in friction, assuming (a) uniform pressure and (b) uniform wear.

■ **Solution**

(a) Uniform pressure

$$T = \left(\frac{2}{3}\right) \cdot \mu W R$$

$$= \left(\frac{2}{3}\right) \times 0.06 \times 30 \times 10^3 \times 0.07$$

$$= 84 \text{ Nm}$$

$$\text{Power lost in friction} = T\omega$$

$$= 84 \times \left(2\pi \times \frac{120}{60}\right) \times 10^{-3}$$

$$= 1.056 \text{ kW}$$

(b) Uniform wear

$$T = \mu W \frac{R}{2}$$

$$= 0.06 \times 30 \times 10^3 \times \frac{0.07}{2}$$

$$= 63 \text{ Nm}$$

$$\text{Power lost in friction} = 63 \times \left(2\pi \times \frac{120}{60}\right) \times 10^{-3}$$

$$= 0.79 \text{ kW}$$

Example 4.5

A conical pivot supports a load of 25 kN, the cone angle being 120°, and the intensity of normal pressure does not exceed 0.25 MPa. The external radius is twice the internal diameter. Find the outer and inner radii of the bearing surface. If the shaft rotates at 180 rpm and the coefficient of friction is 0.15, find the power lost in friction, assuming uniform pressure.

■ **Solution**

Intensity of normal pressure, $p_n = \dfrac{W}{\pi(r_2^2 - r_1^2)}$

$$0.25 = \frac{25 \times 10^3}{\pi\left[(2r_1)^2 - r_1^2\right]}$$

$$r = 103 \text{ mm}$$

$$r_2 = 206 \text{ mm}$$

Frictional torque, $T_f = \left(\dfrac{2}{3}\right) \cdot \mu W \operatorname{cosec}\alpha \left[\dfrac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right]$

$$= \left(\frac{2}{3}\right) \times 0.15 \times 25 \times 10^3 \times \operatorname{cosec} 60^\circ \left[\frac{206^3 - 103^3}{206^2 - 103^2}\right]$$

$$= 693.8 \text{ Nm}$$

Power lost in friction $= T_f \dfrac{\omega}{1000}$

$$= 693.8 \times \frac{2\pi \times \frac{180}{60}}{1000} = 13.08 \text{ kW}$$

Example 4.6

A thrust shaft of a ship has six collars of 600 mm external diameter and 300 mm internal diameter. The total thrust from the propeller is 120 kN. If the coefficient of friction is 0.15 and speed of the engine 100 rpm, find the power lost in friction at the thrust block, assuming (a) uniform pressure, and (b) uniform wear.

■ **Solution**

(a) Uniform pressure

Frictional torque, $T_f = \left(\dfrac{2}{3}\right) \cdot \mu W \left[\dfrac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right]$

$$= \left(\frac{2}{3}\right) \times 0.15 \times 120 \times 10^3 \times \left[\frac{300^3 - 150^3}{300^2 - 150^2}\right]$$

$$= 4200 \text{ Nm}$$

Power lost in friction $= 4200 \times \dfrac{(2\pi \times 100/60)}{1000} = 43.98 \text{ kW}$

(b) Uniform wear

Frictional torque, $T_f = \mu W r_m$

$$= 0.15 \times 120 \times 10^3 \times \frac{(300 + 150)}{2} = 4050 \text{ Nm}$$

Power lost in friction $= 4050 \times \dfrac{(2\pi \times 100/60)}{1000} = 42.41 \text{ kW}$

Exercises

1 A square threaded screw of mean diameter 30 mm and pitch of threads 5 mm is used to lift a load of 15 kN by a horizontal force applied at the circumference of the screw. Find the force required if the coefficient of friction between screw and nut is 0.02.

2 A turnbuckle with right-hand and left-hand single start square threads is used to couple two railway coaches. The pitch of threads is 10 mm over a mean diameter of 30 mm. The coefficient of friction is 0.15. Find the work to be done in drawing the coaches together a distance of 300 mm against a steady load of 2.5 kN.

3 A vertical two-start square threaded screw of 100 mm mean diameter and 20 mm pitch supports a vertical load of 20 kN. The axial thrust on the screw is taken by a collar bearing of 250 mm outside diameter and 100 mm inside diameter. Find the force required at the end of a lever 450 mm long in order to lift and lower the load. The coefficient of friction for the vertical screw and nut is 0.15 and that for the collar bearing is 0.20.

4 The spindle of a screw jack has single start square threads with an outside diameter of 50 mm and a pitch of 10 mm. The spindle moves in a fixed nut. The load is carried on a swivel head but is not free to rotate. The bearing surface of the swivel head has a mean diameter of 60 mm. The coefficient of friction between the nut and the screw is 0.12 and that between the swivel head and the spindle is 0.10. Calculate the load which can be raised by efforts of 125 N each applied at the end of two levers each of effective length 400 mm. Also find the efficiency of the lifting system.

5 A flat foot step bearing 300 mm in diameter supports a load of 8 kN. If the coefficient of friction is 0.1 and speed of the shaft is 80 rpm, find the power lost in friction, assuming (a) uniform pressure, and (b) uniform wear.

6 A vertical pivot bearing of 200 mm diameter has a cone angle of 150°. If the shaft supports an axial load of 25 kN and the coefficient of friction is 0.25, find the power lost in friction when the shaft rotates at 300 rpm, assuming (a) uniform pressure and (b) uniform wear.

7 A vertical shaft supports a load of 30 kN in a conical pivot bearing. The external radius of the cone is three times the internal radius and the cone angle is 120°. Assuming uniform intensity of pressure of 0.40 MPa, Determine the radii of the bearing. If the coefficient of friction between the shaft and bearing is 0.05 and the shaft rotates at 150 rpm, find the power lost in friction.

8 The thrust on the propeller shaft of a marine engine is taken by 8 collars whose external and internal diameters are 650 mm and 400 mm respectively. The thrust pressure is 0.5 MPa and may be assumed uniform. The coefficient of friction between the shaft and collars is 0.04. If the shaft rotates at 120 rpm, find (a) total thrust on the collars and (b) power absorbed by friction at the bearing.

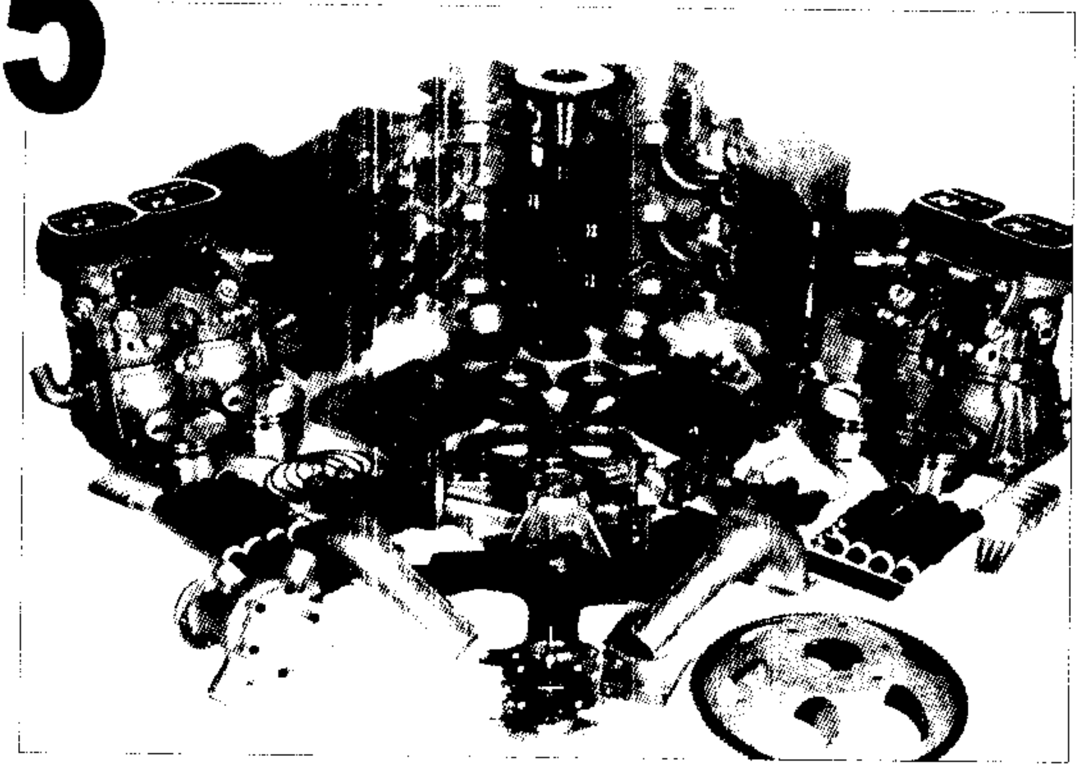
9 The movable jaw of a bench vice is at the upper end of a hinged arm 0.5 m long, the centre line of the screw being 400 mm above the hinge. The screw has 25 mm outside diameter and has 6 mm pitch. The mean radius of the thrust collar is 30 mm. Find the tangential force to be applied to the screw at a radius of 300 mm to produce a force of 6 kN at the jaw. Also find the mechanical efficiency of the vice. Assume the thread and collar coefficients of friction to be 0.1 and 0.15, respectively.

10 A pivot bearing of a shaft consists of a frustrum of a cone. The diameters of the frustrum are 200 mm and 400 mm and its semicone angle is 60°. The shaft carries a load of 40 kN and rotates at 240 rpm. The coefficient of friction is 0.02. Assuming the intensity of pressure to be uniform, determine (a) the magnitude of pressure and (b) the power lost in friction.

11 What force will be required at a radius of 80 mm to raise and lower an 11 kN crossbar of a planer which is raised and lowered by two 38-mm square-thread single-start screws having a pitch of 7 mm? The outside and inside diameters of the collar are 76 mm and 38 mm respectively. Assume the coefficient of friction at the threads as 0.11 and at the collar as 0.13

12 A square-threaded bolt of mean diameter 24 mm and pitch 5 mm is tightened by screwing a nut, whose mean diameter of bearing surface is 50 mm. If the coefficient of friction for nut and bolt is 0.1and that for nut and bearing surface is 0.15, find the force required at the end of a spanner which is 0.2 m long, when the load on the bolt is 12 kN.

5



BELTS, CHAINS AND ROPES

5.1 INTRODUCTION

Belts, chains and ropes are used for power transmission. Belts and chains are used for short centre drives whereas ropes are used for long centre distances. Belts are of two types : flat belts and V-belts. Chains give a more positive drive than belts. There are problems of slip and creep in belts. Belts can be either of the open type or cross type. In this chapter, we shall study belts, chains and ropes from the point of view of power transmission.

5.2 FLAT BELT

5.2.1 Angular Velocity Ratio

Consider the open belt drive shown in Fig.5.1. Let the smaller pulley be the driver and the bigger pulley the driven (or follower). When the driver is rotating anticlockwise, the top side of the belt will be the tight side and the bottom side the slack side.

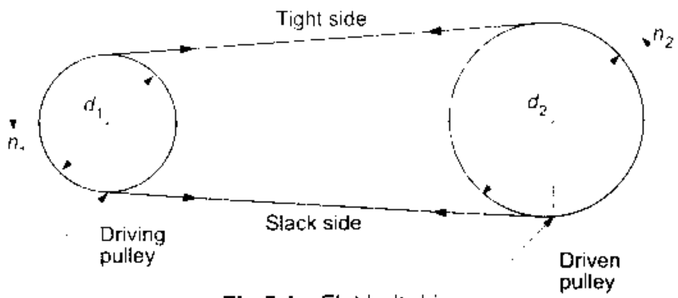


Fig.5.1 Flat belt drive

Let d_1 = diameter of the driver pulley

d_2 = diameter of the driven pulley

n_1 = rpm of the driver

n_2 = rpm of the driven

Then angular velocity of driver, $\omega_1 = \dfrac{2\pi n}{60}$ rad/s

and angular velocity of follower, $\omega_2 = \dfrac{2\pi n_2}{60}$ rad/s

Angular velocity ratio, $\dfrac{\omega_1}{\omega_2} = \dfrac{n_1}{n_2}$

Linear velocity of driver, $v_1 = \pi d_1 n_1$

Linear velocity of follower, $v_2 = \pi d_2 n_2$

Assuming there is no slip between the belt and the pulleys, $v_1 = v_2$.

$$\frac{n_1}{n_2} = \frac{d_2}{d_1} \tag{5.1}$$

If t is the thickness of the belt, then

$$n_1/n_2 = (d_2 + t)/(d_1 + t) \tag{5.2}$$

5.2.2 Effect of Slip

Slip is the difference between the speed of the driver and the belt on the driver side and the belt and the follower on the driven side.

Let s_1 = percentage slip between the driver and the belt

s_2 = percentage slip between the belt and the follower

Linear speed of belt on driver $= v_1\left(\dfrac{1 - s_1}{100}\right)$

Linear speed of follower, $v_2 = v_1\left(\dfrac{1 - s_1}{100}\right)\left(\dfrac{1 - s_2}{100}\right)$

$$= v_1\left[\frac{1 - (s_1 + s_2 - 0.01 s_1 s_2)}{100}\right] = v_1\left(\frac{1 - s}{100}\right)$$

where $s = \dfrac{(s_1 + s_2 - 0.01 s_1 s_2)}{100}$ is the total percentage slip.

Hence
$$\frac{n_2}{n_1} = \left(\frac{1-s}{100}\right)\cdot\frac{d_1+t}{d_2+t} \tag{5.3}$$

5.2.3 Law of Belting

The law of belting states that the centre line of the belt as it approaches the pulley must lie in a plane perpendicular to the axis of that pulley, or must lie in the plane of the pulley, otherwise the belt will run off the pulley.

5.2.4 Length of Open Belt

Consider the open flat belt drive shown in Fig.5.2. The length of open belt.

$$L_o = \text{arc } AEB + \text{arc } CFD + AC + BD$$
$$\text{arc } AEB = \frac{d_1(\pi - 2\alpha)}{2}$$
$$\text{arc } CFD = \frac{d_2(\pi + 2\alpha)}{2}$$
$$AC = BD = O_1G = C\cos\alpha$$
$$L_o = \frac{d_1(\pi - 2\alpha)}{2} + \frac{d_2(\pi + 2\alpha)}{2} + 2C\cos\alpha$$

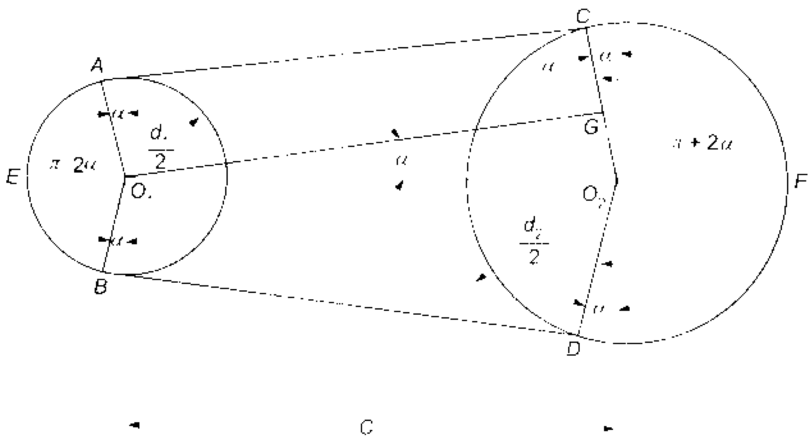


Fig.5.2 Open flat belt drive

Now
$$\sin\alpha = \frac{O_2G}{O_1O_2} = \frac{(O_2C - CG)}{C} = \frac{(O_2C - AO_1)}{C}$$
$$\alpha \approx \sin\alpha = \frac{d_2 - d_1}{2C}$$
$$\cos\alpha = \left[1 - \sin^2\alpha\right]^{0.5} = \left[1 - \left\{\frac{(d_2 - d_1)}{2C}\right\}^2\right]^{0.5}$$
$$\approx 1 - \frac{(d_2 - d_1)^2}{8C^2}$$

$$L_o = \frac{\pi(d_1+d_2)}{2} + \alpha(d_2 - d_1) + 2C\left[1 - \frac{(d_2-d_1)^2}{8C^2}\right]$$

$$L_o = \frac{\pi(d_1+d_2)}{2} + \frac{(d_2-d_1)^2}{2C} + 2C\left[1 - \frac{(d_2-d_1)^2}{8C^2}\right]$$

$$= 2C + \frac{(d_2-d_1)^2}{4C} + \frac{\pi(d_1+d_2)}{2} \qquad (5.4)$$

5.2.5 Length of Cross Belt

Consider the cross belt shown in Fig.5.3. The length of cross flat belt,

$$L_c = \text{arc } AEB + \text{arc } CEB + AD + BC$$

$$\text{arc } AEB = (\pi + 2\alpha)\frac{d_1}{2}$$

$$\text{arc } CEB = (\pi + 2\alpha)\frac{d_2}{2}$$

$$AD = BC = O_2G = C\cos\alpha$$

Now

$$\sin\alpha = \frac{O_1G}{O_1O_2} = \frac{O_1A + AG}{C} = \frac{O_1A + O_2D}{C}$$

$$= \frac{d_1 + d_2}{2C}$$

$$\cos\alpha = \left[1 - \left\{\frac{d_1 + d_2}{2C}\right\}^2\right]^{0.5}$$

$$= 1 - \frac{(d_1+d_2)^2}{8C^2}$$

$$L_c = 2C\left[1 - \frac{(d_1+d_2)^2}{8C^2}\right] + \frac{\pi(d_1+d_2)}{2} + \frac{(d_1+d_2)^2}{2C}$$

$$= 2C + \frac{\pi(d_1+d_2)}{2} + \frac{(d_1+d_2)^2}{4C} \qquad (5.5)$$

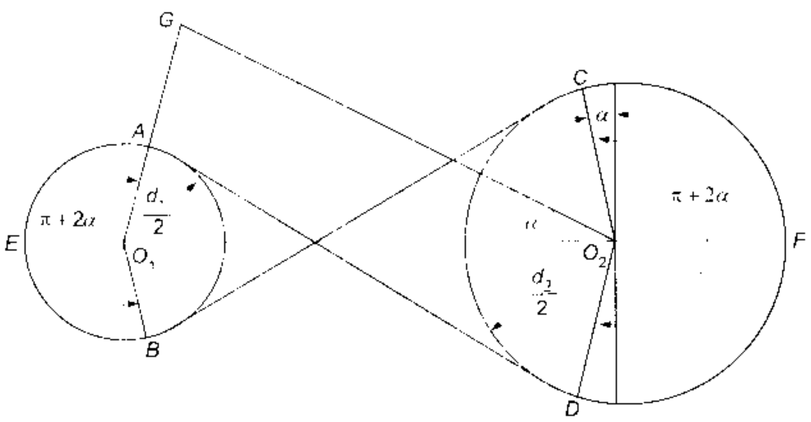


Fig.5.3 Cross flat belt drive

5.2.6 Angle of Arc of Contact

1. *Open belt* The angle of arc of contact on the smaller pulley.

$$\theta = \pi - 2\alpha$$
$$= \pi - 2\sin^{-1}\left[\frac{d_2 - d_1}{2C}\right] \tag{5.6a}$$

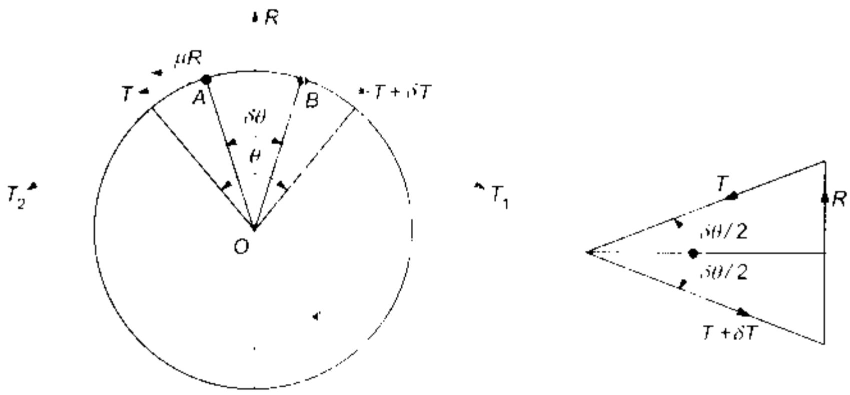
The possibility of slip is more on the smaller pulley due to smaller angle of arc of contact.

2. *Cross belt* The angle of arc of contact on either pulley.

$$\theta = \pi + 2\alpha$$
$$= \pi + 2\sin^{-1}\left[\frac{d_1 + d_2}{2C}\right] \tag{5.6b}$$

5.2.7 Ratio of Belt Tensions

Let T_1 and T_2 be the belt tensions on the tight and slack sides respectively, as shown in Fig.5.4(a). Let θ be the angle of contact on the pulley. Consider a portion AB of the belt on angular arc $\delta\theta$. Let the tension change from T to $T + \delta T$ in going from A to B. Let R be the normal reaction on the pulley and μR the force of friction. The forces acting on the belt-pulley system are shown in Fig.5.4(b).



(a) Forces on the belt (b)

Fig.5.4 Belt tensions

Resolving the forces vertically, we have

$$R = T\sin\left(\frac{\delta\theta}{2}\right) + (T + \delta T)\sin\left(\frac{\delta\theta}{2}\right)$$
$$\approx T\frac{\delta\theta}{2} + (T + \delta T)\frac{\delta\theta}{2}$$
$$\approx T\delta\theta$$

Resolving the forces horizontally, we have

$$\mu R = (T + \delta T)\cos\left(\frac{\delta\theta}{2}\right) - T\cos\left(\frac{\delta\theta}{2}\right) \approx \delta T$$

or $$\mu T\,\delta\theta = \delta T$$

or $$\frac{\delta T}{T} = \mu \cdot \delta\theta$$

$$\int_{T_2}^{T_1} \frac{dT}{T} = \int_0^{\theta} \mu \cdot d\theta$$

$$\ln\left(\frac{T_1}{T_2}\right) = \mu\theta$$

or $$\frac{T_1}{T_2} = \exp(\mu\theta) \tag{5.7}$$

The ratio of belt tensions is given by (5.7).

5.2.8 Power Transmitted

Power transmitted, $$P = \frac{(T_1 - T_2)v}{1000} \text{ kW} \tag{5.8a}$$

$$= T_1 v \frac{[1 - \exp(-\mu\theta)]}{10^3} \text{ kW} \tag{5.8b}$$

5.2.9 Centrifugal Tension

The centrifugal tension is introduced in the belt due to its mass. Let T_c be the centrifugal tension in the belt. Consider the length of the belt over an angular arc $\delta\theta$, as shown in Fig.5.5. The force acting on the belt due to belt tension,

$$= 2T_c \sin\left(\frac{\delta\theta}{2}\right)$$

$$\approx T_c\,\delta\theta \tag{5.8c}$$

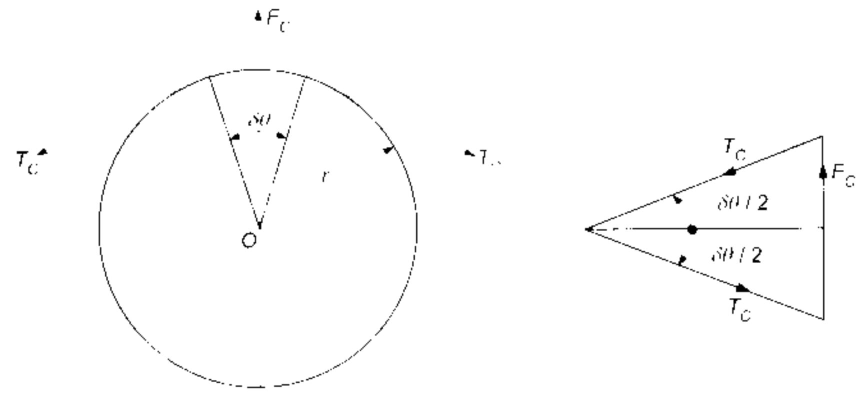


Fig.5.5 Centrifugal tension

Centrifugal force acting on length AB and of unit width of the belt due to angular velocity,

$$F_c = \frac{m \cdot r\delta\theta \cdot 1 \cdot v^2}{r} = mv^2\delta\theta$$

where m = mass of the belt per unit length

r = radius of the pulley

v = speed of belt

For equilibrium of the belt,

$$T_c\delta\theta = mv^2\delta\theta$$

$$T_c = mv^2 \tag{5.9}$$

Centrifugal stress,

$$\sigma_c = \frac{T_c}{bt} = \frac{mv^2}{bt} \tag{5.10}$$

where b = width of the belt and

t = thickness of the belt

Effective tension on tight side $= T_1 - T_c$

Effective tension on slack side $= T_2 - T_c$

5.2.10 Condition for Maximum Power Transmission

Power transmitted,

$$P = (T_1 - T_2)v$$

$$= T_1[1 - \exp(-\mu\theta)]v$$

Let $1 - \exp(-\mu\theta) = k$, so that

$$P = kT_1v$$

Now maximum belt tension,

$$T = T_1 + T_c$$

or

$$T_1 = T - T_c$$

$\therefore$

$$P = k(T - T_c)v$$

$$= kTv - kT_cv$$

$$= kTv - kmv^3$$

For P to be maximum,

$$\frac{dP}{dv} = 0$$

or

$$kT - 3kmv^2 = 0$$

or

$$T = 3mv^2$$

or

$$v = \left[\frac{T}{3m}\right]^{0.5} \tag{5.11}$$

$$P_{max} = kT\left(\frac{T}{3m}\right)^{0.5} - km\left(\frac{T}{3m}\right)^{1.5}$$

$$= k\left(\frac{T}{3m}\right)^{0.5}\left[T - \frac{T}{3}\right]$$

$$= \left(\frac{2}{3}\right)\frac{kT^{1.5}}{(3m)^{0.5}} \qquad (5.12)$$

5.2.11 Initial Belt Tension

Let T_o = initial tension in belt

Resultant tension on tight side = $T_1 - T_o$

Resultant tension on slack side = $T_o - T_2$

Since the belt length remains constant, therefore

$$T_1 - T_o = T_o - T_2$$

or
$$T_o = \frac{T_1 + T_2}{2} \qquad (5.13a)$$

Considering centrifugal tension,
$$T_o = \frac{T_1 + T_2 + 2T_c}{2} \qquad (5.13b)$$

5.2.12 Effect of Initial Tension on Power Transmission

Ratio of tensions,
$$\frac{T_1}{T_2} = \exp(\mu\theta) = e$$

Initial tensions,
$$T_o = \frac{T_1 + T_2 + 2T_c}{2}$$

or
$$T_1 + T_2 = 2(T_o - T_c)$$

$$T_1\left(1 + \frac{1}{e}\right) = 2(T_o - T_c)$$

$$T_1 = \left[\frac{2e}{1+e}\right](T_o - T_c)$$

and
$$T_2 = \left[\frac{2}{1+e}\right](T_o - T_c)$$

$$T_1 - T_2 = \left[\frac{2(e-1)}{e+1}\right](T_o - T_c)$$

Power transmitted,
$$P = (T_1 - T_2)v$$

$$= \left[\frac{2(e-1)}{e+1}\right](T_o - T_c)v$$

$$= \left[\frac{2(e-1)}{e+1}\right]\left(T_o - mv^2\right)v$$

For P to be maximum,

$$\frac{dP}{dv} = 0$$

$$\frac{2(e-1)}{(e+1)}\left(T_o - 3mv^2\right) = 0$$

$$T_o = 3mv^2$$

$$= 3T_c \qquad (5.14)$$

$$v = \left[\frac{T_o}{3m}\right]^{0.5} \qquad (5.15)$$

$$P_{max} = \left[\frac{2(e-1)}{e+1}\right]\left(\frac{T_o - T_o}{3}\right)\left[\frac{T_o}{3m}\right]^{0.5}$$

$$= \left(\frac{4}{3}\right)\left[\frac{e-1}{e+1}\right]\frac{T_o^{1.5}}{(3m)^{0.5}} \qquad (5.16)$$

While starting, $v = 0$ and $T_c = 0$. Hence

$$T_1 = \left[\frac{2e}{e+1}\right]T_o$$

While running,

$$T_1 = \left[\frac{2e}{e+1}\right](T_o - T_c)$$

Hence maximum belt tension,

$$T_1 = \left[\frac{2e}{e+1}\right]T_o \qquad (5.17)$$

5.2.13 Belt Creep

The tension on the tight side is more than the tension on the slack side. As a result of this, the belt is stretched more on the tight side as compared to the slack side. Therefore, the driver pulley receives more length of the belt and delivers less. Hence, the belt creeps forward. The reverse occurs on the follower pulley. The follower pulley receives less length of the belt and delivers more. As a result of it, the belt creeps backward. This phenomenon is called *creeping of the belt*.

$$\text{Creep} = \frac{T_1 - T_2}{btE} \qquad (5.18)$$

where E is the modulus of elasticity of belt material.

The velocity ratio becomes,

$$\frac{n_2}{n_1} = \left(\frac{d_1}{d_2}\right)\left[\frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}\right] \qquad (5.19)$$

5.2.14 Crowning of Pulleys

The pulley face is given a convex curvature and is never kept flat. This is called crowning of the pulleys. This helps in running the belt in the centre of the pulley width. Crowning prevents any tendency of the belt to fall off the pulley face.

5.2.15 Cone Pulleys

Consider the cone pulley block shown in Fig.5.6. Let N be the speed of the driver block, and R_1, R_2, R_3, etc. the radii of its pulleys. Let the radii of the driven pulleys be r_1, r_2, r_3, etc. and speeds n_1, n_2, n_3, etc. The centre distance between the pulleys is C.

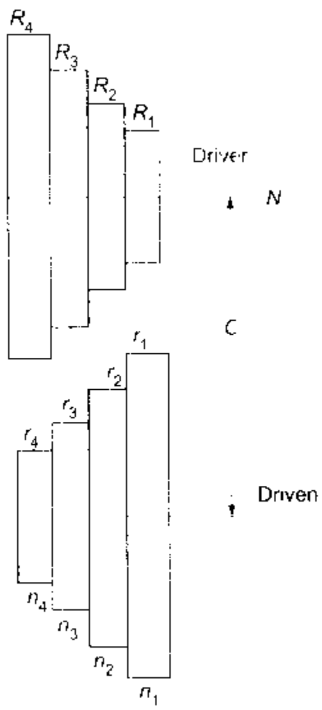


Fig.5.6 Cone pulley system

Now $R_1 N = r_1 n_1$

or $\frac{N}{n_1} = \frac{r_1}{R_1}$

Similarly $\frac{N}{n_2} = \frac{r_2}{R_2}$

$\frac{N}{n_3} = \frac{r_3}{R_3}$ and so on.

Let $n_2 = k n_1$

Therefore $\frac{n_2}{N} = \frac{k n_1}{N} = k\left(\frac{R_1}{r_1}\right)$

Similarly $\frac{n_3}{N} = k^2\left(\frac{R_1}{r_1}\right)$

or in general, $\frac{n_i}{N} = k^{(i-1)}\left(\frac{R_1}{r_1}\right) = \frac{R_i}{r_i}$ (5.20)

and $\frac{n_i}{n_1} = k^{(i-1)}$ (5.21)

The length of the belt is same for all the cones, therefore

$$R_i + r_i = \text{constant}, \quad i = 1, 2, 3 \quad \text{etc.} \tag{5.22}$$

5.2.16 Compound Belt Drive

In compound belt drive, the driven pulley of the first set is mounted on the same shaft on which the driver of the second set is mounted. Let pulley 1 be the driver for the first set and pulley 2 its follower. The driver of the second set, pulley 3, is mounted on the same shaft on which pulley 2 is mounted. The follower of second set is pulley 4.

$$\frac{n_2}{n_1} = \frac{d_1}{d_2}$$

$$\frac{n_4}{n_3} = \frac{d_3}{d_4}$$

Hence

$$\frac{n_4}{n_1} = \left(\frac{d_1 d_3}{d_2 d_4}\right)$$

or in general,

$$\frac{\text{Speed of last driven}}{\text{Speed of first driver}} = \frac{\text{Product of diameters of drivers}}{\text{Product of diameters of drivens}} \tag{5.23}$$

Example 5.1

Two pulleys of diameters 450 mm and 150 mm are mounted on two parallel shafts 2 m apart and are connected by a flat belt drive. Find the power which can be transmitted by the belt when the larger pulley rotates at 180 rpm. The maximum permissible tension in the belt is 1 kN, and the coefficient of friction between the belt and the pulley is 0.25. Also find the length of the cross belt required and the angle of arc of contact between the belt and the pulleys.

■ **Solution**

$$v_2 = \frac{\pi \times 450 \times 180}{60 \times 1000} = 4.24 \text{ m/s}$$

$$n_1 = \frac{180 \times 450}{150} = 540 \text{ rpm}$$

Now

$$\sin\alpha = \frac{d_1 + d_2}{2C} = \frac{150 + 450}{4000} = 0.15$$

$$\alpha = 8.627°$$

Angle of contact,

$$\theta = \pi + 2\alpha = 197.254° \quad \text{or} \quad 3.443 \text{ rad}$$

$$\frac{T_1}{T_2} = \exp(\mu\theta)$$

$$= \exp(0.25 \times 3.443) = 2.365$$

$$T_2 = \frac{T_1}{2.365} = \frac{1000}{2.365} = 422.83 \text{ N}$$

Power transmitted,

$$P = (T_1 - T_2)\frac{v_2}{1000}$$

$$= (1000 - 422.83) \times \frac{4.24}{1000} = 2.447 \text{ kW}$$

Length of cross belt,

$$L_c = 2C + \frac{\pi(d_1 + d_2)}{2} + \frac{(d_1 + d_2)^2}{4C}$$

$$= 4 + \frac{\pi(0.450 + 0.150)}{2} + \frac{(0.450 + 0.150)^2}{8}$$

$$= 4.987 \text{ m}$$

Example 5.2

A shaft running at 200 rpm drives another shaft at 400 rpm, and transmits 7.5 kW through an open belt. The belt is 80 mm wide and 10 mm thick. The centre distance is 4 m. The smaller pulley is of 500 mm diameter, and the coefficient of friction between the belt and pulley is 0.30. Calculate the stress in the belt.

■ **Solution**

$$d_2 = \frac{n_1 d_1}{n_2} = 400 \times \frac{500}{200} = 1000 \text{ mm}$$

$$v_1 = \frac{\pi \times 500 \times 400}{60 \times 1000} = 10.472 \text{ m/s}$$

$$\sin\alpha = \frac{d_2 - d}{2C} = \frac{1000 - 500}{8000} = 0.0625$$

$$\alpha = 3.583°$$

Angle of arc of contact, $\theta = 180° - 2\alpha = 172.83° = 3.0165$ rad

$$\frac{T_1}{T_2} = \exp(\mu\theta) = \exp(0.3 \times 3.0165) = 2.472 \quad (1)$$

Power transmitted, $P = (T_1 - T_2)\dfrac{v}{1000}$ kW

$$(T_1 - T_2) = \frac{7.5 \times 1000}{10.472} = 716.2 \text{ N} \quad (2)$$

From (1) and (2), we get

$$T_1 = 1202.7 \text{ N} \quad \text{and} \quad T_2 = 486.5 \text{ N}$$

Maximum stress in the belt $= \dfrac{T_1}{bt}$

$$= \frac{1202.7}{80 \times 10} = 1.503 \text{ N/mm}^2$$

Example 5.3

A leather belt is required to transmit 8 kW from a pulley 1.5 m diameter running at 240 rpm. The angle of contact is 160° and the coefficient of friction between belt and pulley is 0.25. The safe working stress for leather is 1.5 MPa and density of leather is 1000 kg/m^3. Determine the width of the belt if its thickness is 10 mm. Take into account the effect of centrifugal tension.

■ **Solution**

Velocity of the belt, $v = \dfrac{\pi \times 1.5 \times 240}{60} = 18.85$ m/s

Power transmitted, $P = (T_1 - T_2)\dfrac{v}{1000}$

$$T_1 - T_2 = 8 \times \frac{1000}{18.85} = 424.4 \text{ N}$$

$$\frac{T_1}{T_2} = \exp(\mu\theta)$$

$$= \exp\left(0.25 \times 160 \times \frac{\pi}{180}\right) = 2.01$$

$$T_1 = 844.6 \text{ N} \quad \text{and} \quad T_2 = 420.2 \text{ N}$$

Mass of the belt per metre length, $m = b \times 0.01 \times 1 \times 1000 = 10b$ kg

Centrifugal tension, $T_c = mv^2 = 10b(18.85)^2 = 3553.2 \times b$ N

Maximum tension in the belt, $T = \sigma \cdot bt$

$$= 1.5 \times 10^6 \times b \times 0.01 = 15000 \times b \text{ N}$$

$$T = T_1 + T_c$$

or $$15000\,b = 844.6 + 3553.2b$$

$$b = 0.0738 \text{ m} \quad \text{or} \quad 73.8 \text{ mm}$$

Example 5.4

A pulley is driven by a flat belt 100 mm wide and 6 mm thick. The density of belt material is 1000 kg/m^3. The angle of lap is 120° and the coefficient of friction 0.3. The maximum stress in the belt does not exceed 2 MPa. Find the maximum power that can be transmitted and the corresponding speed of the belt.

■ **Solution**

Maximum tension in the belt, $T = 2 \times 10^6 \times 0.1 \times 0.006 = 1200$ N

Mass of the belt per metre length, $m = 0.1 \times 0.006 \times 1 \times 1000 = 0.6$ kg/m

Speed of the belt for maximum power, $v = \left[\dfrac{T}{3m}\right]^{0.5}$

$$= \left[\frac{1200}{3 \times 0.6}\right]^{0.5}$$

$$= 25.82 \text{ m/s}$$

For maximum power to be transmitted, the centrifugal tension,

$$T_c = \frac{T}{3} = \frac{1200}{3} = 400 \text{ N}$$

$$\frac{T_1}{T_2} = \exp(0.3 \times 120 \times \pi/180) = 1.874$$

$$T_1 = T - T_c = 1200 - 400 = 800 \text{ N}$$

$$T_2 = \frac{T_1}{1.874} = \frac{800}{1.874} = 426.8 \text{ N}$$

Maximum power transmitted $= (T_1 - T_2)\dfrac{v}{1000}$

$$= (800 - 426.8) \times \frac{25.82}{1000}$$

$$= 9.636 \text{ kW}$$

Example 5.5

An open belt drive is used to connect two parallel shafts 4 m apart. The diameter of bigger pulley is 1.5 m and that of the smaller pulley 0.5 m. The mass of the belt is 1 kg/m length. The maximum tension is not to exceed 1500 N. The coefficient of friction is 0.25. The bigger pulley, which is the driver, runs at 250 rpm. Due to slip, the speed of the driven pulley is 725 rpm. Calculate the power transmitted, power lost in friction, and the efficiency of the drive.

Solution

$$v_1 = \pi \times 1.5 \times \frac{250}{60} = 19.635 \text{ m/s}$$

$$v_2 = \pi \times 0.5 \times \frac{725}{60} = 18.98 \text{ m/s}$$

$$T_c = mv_1^2 = 1 \times (19.635)^2 = 385.53 \text{ N}$$

$$T_1 = T - T_c = 1500 - 385.53 = 1114.47 \text{ N}$$

$$\sin\alpha = \frac{d_2 - d_1}{2C} = \frac{1.5 - 0.5}{8} = 0.125$$

$$\alpha = 7.18°$$

$$\theta = 180° - 2\alpha = 165.64° = 2.89 \text{ rad}$$

$$\frac{T_1}{T_2} = \exp(\mu\theta) = \exp\ (0.25 \times 2.89) = 2.06$$

$$T_2 = \frac{T_1}{2.06} = \frac{1114.47}{2.06} = 541.0 \text{ N}$$

$$T_1 - T_2 = 1114.47 - 541.00 = 573.47 \text{ N}$$

Torque on bigger pulley, $M_1 = 573.47 \times 0.75 = 430.1$ Nm

Torque on smaller pulley, $M_2 = 573.47 \times 0.25 = 143.37$ Nm

Power transmitted, $P = (T_1 - T_2)\dfrac{v_1}{1000} = 573.47 \times \dfrac{19.635}{1000} = 11.26$ kW

Input power, $P_1 = M_1\omega_1 = 430.1 \times \dfrac{\left(2\pi \times \frac{250}{60}\right)}{1000} = 11.26$ kW

Output power, $P_2 = M_2\omega_2 = 143.37 \times \dfrac{\left(2\pi \times \frac{725}{60}\right)}{1000} = 10.885$ kW

Power lost in friction, $P_f = 11.26 - 10.885 = 0.375$ kW

Efficiency of the drive $= \dfrac{10.885}{11.26} = 96.67\%$

Example 5.6

Two parallel shafts 5 m apart are connected by open flat belt drive. The diameter of the bigger pulley is 1.5 m and that of the smaller pulley 0.75 m. The initial tension in the belt is 2.5 kN. The mass of the belt is 1.25 kg/m length and coefficient of friction is 0.25. Taking centrifugal tension into account, find the power transmitted, when the smaller pulley rotates at 450 rpm.

Solution

$$v = \frac{\pi \times 0.75 \times 450}{60} = 17.67 \text{ m/s}$$

$$T_c = mv^2 = 1.25 \times (17.67)^2 = 390.28 \text{ N}$$

$$T_o = \frac{T_1 + T_2 + 2T_c}{2}$$

$$2500 = \frac{T_1 + T_2 + 2 \times 390.28}{2}$$

$$T_1 + T_2 = 4219.44 \text{ N}$$

$$\alpha = \sin^{-1}\left[\frac{1.5-0.75}{10}\right] = 4.3°$$

$$\theta = 180° - 2\alpha = 171.4° = 2.991 \text{ rad}$$

$$T_1/T_2 = \exp(\mu\theta) = \exp(0.25 \times 2.991) = 2.1125$$

$$T_2 = 1355.65 \text{ N} \quad \text{and} \quad T_1 = 2863.81 \text{ N}$$

Power transmitted,
$$P = (T_1 - T_2)\frac{v}{1000}$$

$$= (2863.81 - 1355.65) \times \frac{17.67}{1000} = 26.65 \text{ kW}$$

5.3 V-BELT DRIVE

The V-belt drive is more positive than the flat belt drive. It is a short centre drive and is preferred for power transmission from the prime mover. The belt touches the sides of the grooved pulley only. The V-belts are classified as: *A, B, C, D, E.*

5.3.1 Ratio of Belt Tensions

A V-belt in a grooved pulley is shown in Fig.5.7.

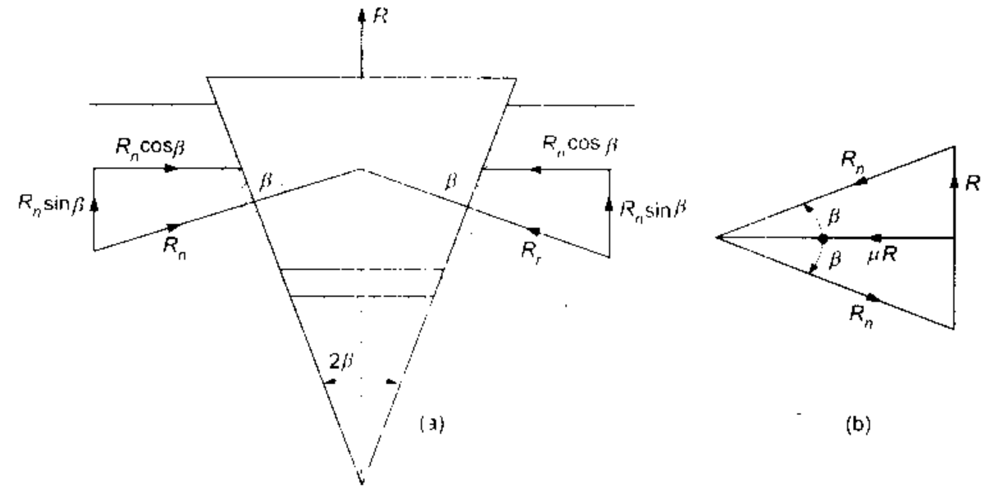


Fig.5.7 Forces on V-belt

Let
2β = pulley groove angle;
R = total reaction on the pulley;
R_n = normal reaction between the belt and the sides of the groove and
μ = coefficient of friction between the belt and the groove sides.

Then
$$R = 2R_n \sin\beta$$

or
$$R_n = \frac{R}{2\sin\beta}$$

Force of friction,
$$F = 2\mu R_n = \left(\frac{\mu}{\sin\beta}\right)R = \mu_e R$$

where μ_e is called the virtual, apparent or equivalent coefficient of friction.

Ratio of tensions, $$\frac{T_1}{T_2} = \exp(\mu_e \theta) \qquad (5.24)$$

Example 5.7

A compressor requires 100 kW to run at 240 rpm from an electric motor of speed 750 rpm, by means of a V-belt drive. The diameter of the compressor shaft pulley should not be more than 1 m while the centre distance between the shafts is 2 m. The belt speed should not exceed 25 m/s.

Determine the number of V-belts required to transmit the power if each belt has a cross-sectional area of 375 mm^2, density 1000 kg/m^3, and an allowable tensile stress of 2.5 MPa. The pulley groove angle is 40° and coefficient of friction between the belt and the pulley sides is 0.25.

■ **Solution**

Diameter of motor pulley, $$d_1 = \frac{240 \times 1}{750} = 0.32 \text{ m}$$

Mass of belt per metre length, $$m = 375 \times 10^{-6} \times 1 \times 1000 = 0.375 \text{ kg/m}$$

Velocity of belt, $$v = 25 \text{ m/s}$$

Centrifugal tension, $$T_c = mv^2 = 0.375 \times 625 = 234.375 \text{ N}$$

Maximum tension in the belt, $$T = \sigma A = 2.5 \times 10^6 \times 375 \times 10^{-6} = 937.5 \text{ N}$$

Tight side tension, $$T_1 = T - T_c = 937.5 - 234.375 = 703.125 \text{ N}$$

$$\sin\alpha = \frac{d_2 - d_1}{2C} = \frac{1 - 0.32}{4} = 0.17$$

$$\alpha = 9.78°$$

$$\theta = 180° - 2\alpha = 160.44° = 2.8 \text{ rad}$$

$$T_1/T_2 = \exp(\mu_e \theta) = \exp\left[\left(\frac{0.25}{\sin 20°}\right) \times 2.8\right] = \exp(2.0466) = 7.74$$

$$T_2 = \frac{703.125}{7.74} = 90.84 \text{ N}$$

Power transmitted per belt $$= (T_1 - T_2)\frac{v}{1000} = (703.125 - 90.84) \times \frac{25}{1000} = 15.31 \text{ kW}$$

Number of V-belts required $$= \frac{100}{15.31} = 6.53 \approx 7$$

5.4 CHAIN DRIVE

Chains are mostly used to transmit power without slipping and with better efficiency than belts. They are commonly used in motor cycles, bicycles, road rollers, and agricultural machinery. A chain on the sprocket is shown in Fig.5.8. The pitch of the chain is the distance between the hinge centers of the adjacent links. The pitch circle diameter is the diameter of the circle on which the hinge centers of the chain link lie, when the chain is wrapped round the sprocket.

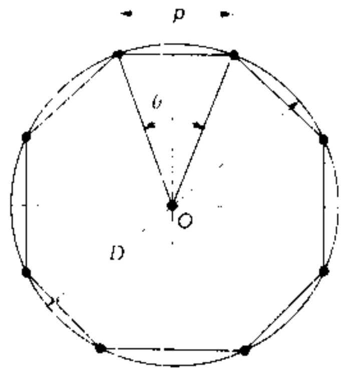


Fig.5.8 Chain pitch

5.4.1 Chain Pitch

As shown in Fig.5.8, the chain pitch is,

$$p = \frac{2 \times D}{2} \times \sin\left(\frac{\theta}{2}\right) = D \sin\left(\frac{\theta}{2}\right) \qquad (5.25a)$$

where $\theta = \frac{360°}{z}$ and z = Number of teeth

Hence,

$$p = D \sin\left(\frac{180°}{z}\right) \qquad (5.25b)$$

5.4.2 Chain Length

As shown in Fig.5.9, the chain length

$$L = 2C + \pi (R_1 + R_2) + \frac{(R_1 - R_2)^2}{C} \qquad (5.26a)$$

$$\pi (R_1 + R_2) = \frac{p (z_1 + z_2)}{2}$$

$$R_1 = \left(\frac{p}{2}\right) \operatorname{cosec}\left(\frac{180°}{z_1}\right)$$

$$R_2 = \left(\frac{p}{2}\right) \operatorname{cosec}\left(\frac{180°}{z_2}\right)$$

$$L = 2C + \frac{p (z_1 + z_2)}{2} + \frac{p^2 \left[\operatorname{cosec}\left(\frac{180°}{z_1}\right) - \operatorname{cosec}\left(\frac{180°}{z_2}\right)\right]^2}{4C} \qquad (5.26b)$$

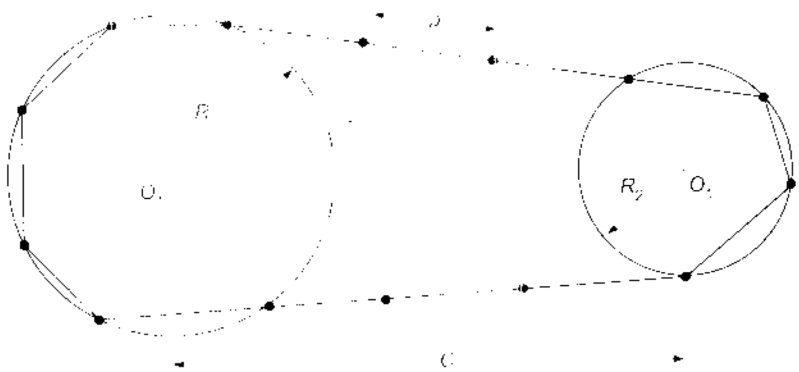


Fig.5.9 Chain drive

Example 5.8

A chain drive is used for reduction of speed from 240 rpm to 120 rpm. The number of teeth on the driving sprocket is 24. Find the number of teeth on the driven sprocket. If the pitch circle diameter of the driven sprocket is 600 mm and centre distance is 1 m, determine the pitch and length of the chain.

■ **Solution**

$$z_2 = \frac{n_1 z_1}{n_2} = \frac{240 \times 24}{120} = 48$$

$$R_2 = \left(\frac{p}{2}\right)\text{cosec}\left(\frac{180°}{z_2}\right)$$

$$0.3 = \left(\frac{p}{2}\right)\text{cosec}\left(\frac{180°}{48}\right)$$

$$p = 0.0392 \text{ m} \quad \text{or} \quad 39.2 \text{ mm}$$

$$L = 2C + \frac{p(z_1 + z_2)}{2} + \left(\frac{p^2}{4C}\right)\left[\text{cosec}\left(\frac{180°}{z_1}\right) - \text{cosec}\left(\frac{180°}{z_2}\right)\right]^2$$

$$= 2 \times 1 + \left(\frac{0.0392}{2}\right)(24 + 48) + \left(\frac{0.0392^2}{4}\right)\left[\text{cosec}\left(\frac{180°}{24}\right) - \text{cosec}\left(\frac{180°}{48}\right)\right]^2$$

$$= 2 + 2.8221 + 0.0223 = 4.8447 \text{ m}$$

5.5 ROPES

Ropes are used for power transmission over long distances. They are commonly used in hoisting equipment, drilling rigs, and textile industry. Ropes are either made of fibre or steel. Ropes are generally of circular cross section and require grooved sheaves or pulleys.

5.5.1 Ratio of Tensions

The ratio of tensions, $$\frac{T_1}{T_2} = \exp(\mu\theta) \quad (5.27)$$

where $\mu_e = \dfrac{\mu}{\sin\beta}$

β = semi-groove angle of the sheave

θ = angle of contact.

Example 5.9

A pulley of groove angle 45°, diameter 4 m and having 15 grooves is used to transmit power. The angle of contact is 170° and the coefficient of friction between the ropes and the groove sides is 0.30. The maximum possible tension in the ropes is 1 kN and the mass of the rope is 1.5 kg/m length. Determine the speed of the pulley for maximum power conditions.

■ **Solution**

For maxim power to be transmitted, $v = \left[\dfrac{T}{3m}\right]^{0.5} = \left[\dfrac{1000}{3 \times 1.5}\right]^{0.5} = 14.91$ m/s

Speed of the pulley, $n = \dfrac{60 \times 14.91}{4\pi} = 71.2$ rpm

For maximum power to be transmitted, $T_c = \dfrac{T}{3} = \dfrac{1000}{3} = 333.3$ N

Tension on the side, $T_1 = T - T_c = 1000 - 333.33 = 666.67$ N

$$\frac{T_1}{T_2} = \exp\,(\mu_e\theta)$$

$$= \exp\left[0.3 \times \text{cosec}\,(22.5°) \times 170 \times \frac{\pi}{180}\right]$$

$$= \exp\,(2.326) = 10.23$$

$$T_2 = \frac{T_1}{10.23} = \frac{666.67}{10.23} = 65.12 \text{ N}$$

Power transmitted per rope $= (T_1 - T_2)\dfrac{v}{1000}$

$$= (666.67 - 65.12) \times \frac{14.91}{1000} = 8.97 \text{ kW}$$

Total power transmitted, $P = 8.97 \times 15 = 134.5$ kW

Exercises

1 A flat belt is required to transmit 20 kW from a pulley 1.5 m diameter running at 300 rpm. The angle of contact between the belt and the pulley is 160° and the coefficient of friction is 0.25. The safe working stress for the belt material is 3 MPa. The thickness of belt is 6 mm and its density is 1100 kg/m^3. Find the width of the belt required.

2 A shaft running at 200 rpm carries a pulley 1.25 m diameter which drives a dynamo at 1200 rpm by means of a belt 6 mm thick. Allowing for the thickness of the belt and a slip of 4%, find the size of the dynamo pulley. Also find the width of belt required to transmit 15 kW if the ratio of belt tensions is 2.5. The maximum tension in the belt is not to exceed 3 MPa.

3 A belt is required to transmit 40 kW from a pulley 1.5 m diameter running at 300 rpm. The angle of contact is spread over 11/24 th of the circumference of the pulley, and the coefficient of friction is 0.3. Determine the width of the belt required, if thickness of belt is 10 mm. The safe working stress for belt material is 2.5 MPa and the density of belt material is 1100 kg/m^3.

4 A V-belt having a lap angle of 180o has a cross-sectional area of 250 mm2, and runs in a groove of included angle 40°. The density of the belt material is 1500 kg/m^3 and maximum stress is limited to 4 MPa. The coefficient of friction is 0.15. Find the maximum power that can be transmitted, if the wheel has a mean diameter of 300 mm and runs at 900 rpm.

5 A machine which is to rotate at 400 rpm is run by an engine turning at 1500 rpm, through a silent chain, having a pitch of 15 mm. The number of teeth on a sprocket should be from 18 to 105. The linear velocity of chain drive is not to exceed 10 m/s. Find the suitable number of teeth for both the sprockets.

6 A rope drive transmits 120 kW at 225 rpm by ropes, each 25 mm diameter and density 6800 kg/m^3. The maximum rope tension is 1.5 kN and it is designed for maximum power conditions. The angle of contact is 160° and coefficient of friction is 0.25, determine the diameter of pulley and number of ropes, if the groove angle is 45°.

7 An open belt 100 mm wide connects two pulleys mounted on parallel shafts with their centres 2.5 m apart. The pulleys are of 500 mm and 250 mm diameters. The coefficient of friction between the belt and the pulleys is 0.3. The maximum stress in the belt is limited to 15 N/mm width. Find the maximum power which can be transmitted if the larger pulley rotates at 120 rpm.

8 A leather belt 120 mm wide and 6 mm thick transmits power from a pulley 800 mm diameter which rotates at 450 rpm. The angle of lap is 160° and the coefficient of friction is 0.3. The mass of the belt is 1000 kg/m^3 and the stress is not to exceed 2.5 MPa. Find the maximum power that can be transmitted.

9 An open belt drive connects two pulleys 1.5 m and 0.5 m diameter on parallel shafts 3.5 m apart. The belt has a mass of 1 kg/m length and the maximum tension in the belt is not to exceed 2 kN. The 1.5 m pulley, which is the driver, runs at 250 rpm. Due to belt slip, the velocity of the driven shaft is only 730 rpm. If the coefficient of friction between the belt and the pulley is 0.25, find (a) the torque on each shaft, (b) the power transmitted, (c) the power lost in friction and (d) the efficiency of the drive.

10 The power transmitted between two shafts 4 m apart by a cross belt drive is 7.5 kW. The pulleys are 600 mm and 300 mm diameters and the bigger pulley is the driver, and running at 225 rpm. The permissible load on the belt is 25 N/mm width of the belt, which is 5 mm thick. The coefficient of friction is 0.35. Determine (a) the length of the belt, (b) the width of the belt and (c) the initial tension in the belt.

11 A V-belt drive consists of three belts in parallel on grooved pulleys of the same size. The angle of groove is 40° and the coefficient of friction 0.15. The cross-sectional area of each belt is 800 mm^2 and the permissible stress in the belt material is 3 MPa. Calculate the power that can be transmitted between two pulleys 400 mm in diameter rotating at 960 rpm.

12 A rope drive is required to transmit 250 kW from a sheave of 1 m diameter running at 450 rpm. The safe pull in each rope is 800 N and the mass of the rope is 0.46 kg/m length. The angle of lap is 160° and the groove angle is 45°. If the coefficient of friction between the rope and the sheave is 0.3, find the number of ropes required.

13 The reduction of speed from 360 rpm to 120 rpm is desired by the use of a chain drive. The driving sprocket has 18 teeth. Find the number of teeth on the driven sprocket and the pitch length of the chain, if the pitch radius of the driven sprocket is 250 mm and the centre distance between the two sprockets is 400 mm.

14 A leather belt 150 mm wide 6 mm thick and weighing 6 N/m connects two pulleys each 1 m in diameter and on parallel shafts. The belt is found to slip when the moment of resistance is 600 N m and the speed is 500 rpm. If the coefficient of friction between the belt and the pulleys is 0.24, find the largest tension in the belt.

15 What is the effect of centrifugal force on the transmission of power in a belt drive? A prime mover running at 300 rpm drives a D.C. generator at 500 rpm by a belt drive. Diameter of the pulley on the output shaft of the prime mover is 600 mm. Assuming a slip of 3%, determine the diameter of the generator pulley if the belt running over it is 6 mm thick.

16 An 8-mm-thick leather open belt connects two pulleys. The smaller pulley is 300 mm diameter and runs at 200 rpm. The angle of lap of this pulley is 160°, and the coefficient of friction between the belt and the pulley is 0.25. The belt is on the point of slipping when 3 kW is transmitted. Safe working stress in the belt material is 1.6 N/mm^2. Determine the required width of the belt for 20% overload capacity. Initial tension may be taken equal to mean of the driving tensions. It is proposed to increase the power transmitting capacity of the drive by adopting one of the following alternatives:

(a) increasing the initial tension by 10%.

(b) increasing the coefficient of friction to 0.3 by applying a dressing to the belt.

Examine the two alternatives and recommend the one which you think will be more effective. How much power would the drive transmit adopting either of the alternatives?

17 A rope drive transmits 600 kW from a pulley of effective diameter 4 m, which runs at a speed of 90 rpm. The angle of lap is 160°, the angle of groove 45°, the coefficient of friction 0.28, the weight of the rope 15 N/m and the allowable tension in each rope 2.4 kN. Find the number of ropes required.

18 An electric motor is to drive a compressor by a belt drive.

Power to be transmitted = 7.5 kW

Diameter of motor pulley = 200 mm

Centre distance between pulleys = 1 m

Motor speed = 750 rpm

Compressor speed = 250 rpm

Direction of rotation of both the pulleys is same. Find the width of the belt required if the permissible belt tension is 16 N/mm belt width. Coefficient of friction between the belt and the pulleys is 0.3. Neglect the effect of centrifugal tension.

19 A blower is driven by an electric motor through a belt drive. The motor runs at 750 rpm. For this power transmission a flat belt of thickness 8 mm and width 250 mm is used. The diameter of the motor pulley is 350 mm and that of the blower pulley 1350 mm. The centre distance between these pulleys is 1350 mm and an open belt configuration is adopted. The pulleys are made out of cast iron. Frictional coefficient between belt and pulley is 0.35 and the permissible stress for the belt material can be taken as 2.5 N/mm^2 with sufficient factor of safety. The belt weighs 20 N/m. What is the maximum power transmitted without belt slipping on any one of the pulleys?

20 Determine the width of a 9.75-mm-thick leather belt required to transmit 15 kW from a motor running at 900 rpm. The diameter of the driving pulley of the motor is 300 mm. The driven pulley runs at 300 rpm and the distance between the centre of the two pulleys is 3 m. The weight of the leather is 0.1×10^{-3} N/mm^2. The maximum allowable stress in the leather is 2.5 N/mm^2. The coefficient of friction between leather and pulley is 0.3. Assume open belt drive and neglect the sag and slip of the belt.

21 A prime mover running at 300 rpm, drives a DC generator at 500 rpm by a belt drive. The diameter of the pulley on the output shaft of the prime mover is 600 mm. Assuming a slip of 3%, determine the diameter of the generator pulley if the belt running over it is 6 mm thick.

22 A V-belt of 6 cm^2 cross-section has a groove angle of 40° and angle of lap of 150°, $\mu = 0.1$. The mass of belt per metre run is 1.2 kg. The maximum allowable stress in the belt is 850 N/cm^2. Calculate the power that can be transmitted at a belt speed of 30 m/s.